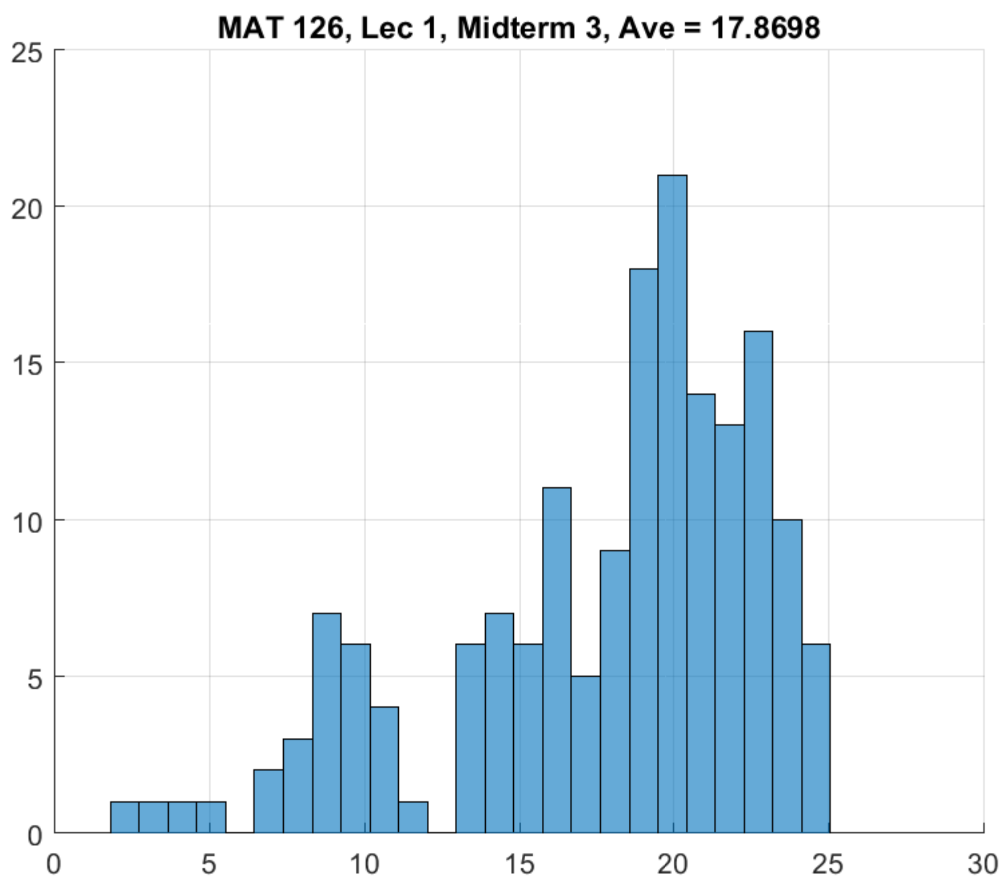
MAT 126.01, Prof. Bishop, Tuesday, Dec 1, 2020
Course grades
Section 7.3 Polar coordinates
Brief introduction to complex numbers


MAT 126, Lec 1, Midterm 3, Ave = 17.8698
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20
15
10
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0
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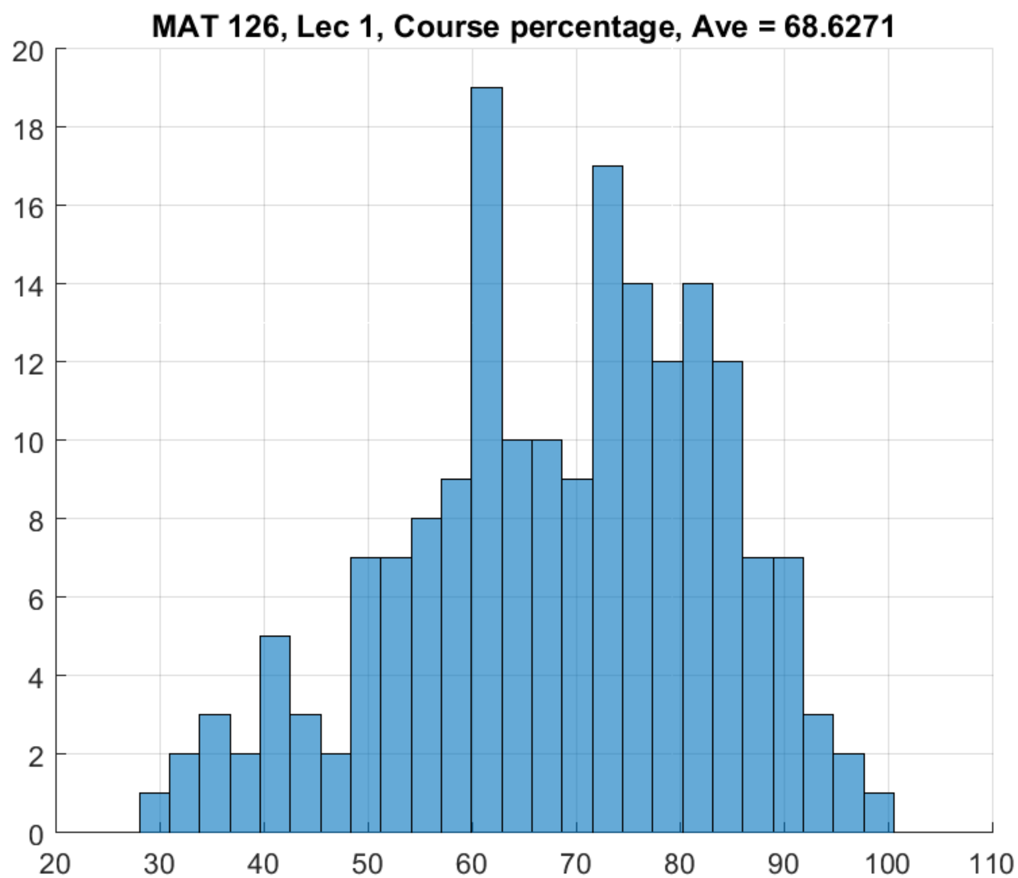
25
30


MAT 126, Lec 1, Course percentage, Ave = 68.6271
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2
4
6
8
10
12
14
16
18
20
20
30
40
50
60
70
80
90
100
110

Grade	Percentage
A	83-100
A-	81-82
B+	78-80
B	65-78
B-	60-64
C+	55-59
C	45-54
D	30-44
F	0-29

Some students have adjusted totals due to late adding, or illness.

If you have questions about your grade email me.

If you don't take the final exam, the tentative grade posted in Blackboard will become your course grade.

If you do take the final exam, I will recompute your total using the final and HW 13. You will get the higher of the two grades.

If your current percentage is $P\%$, a perfect final can raise it to at most $(.9)P + 10$. So if you have a 60% currently it could go to 64%.

Final is online in Lumen and multiple choice. Open book.

I will open a class meeting at 2pm on Dec 10. You must join the class and have your camera on to get credit for the exam.

There are some practice exams posted in Lumen to show you what final will look like. These practices allow repeated attempts, which will not happen on final.

Section 7.3: Polar coordinates

Usual (Cartesian or rectangular) coordinates describe a point by givng x and y coordinates.

Polar coordinates describe point using $r =$ distance from origin, and $\theta =$ angle from postive real axis.

$$x = r\cos\theta, \quad y = r\sin\theta.$$

Convert these points to polar coordinates: $(1, 1)$ and $(-3, 4)$.

Convert these points to rectangular coordinates: $(1, \pi/2)$ and $(3, \pi/4)$.

Plot the curve $r = \sin\theta$.

Plot some example in `MATLAB`

Quick introduction to complex numbers.

Points in plane can be added, substracted, multiplied and divided.

$$(x, y) = x + iy = x + yi$$

where $i^2 = -1$. x is called "real part" and y is called "imaginary part".

$$(a + ib) + (c + id) = (a + b) + i(c + d),$$

$$\begin{aligned}(a + ib) \cdot (c + id) &= ac + iad + ibc + i^2bd \\ &= ac + iad + ibc - bd \\ &= (ac - bd) + i(ad + bc)\end{aligned}$$

$$(2+4i)+(3-2i)=$$

$$4+(2+2i)=$$

$$(1+i)\cdot(2+2i)=$$

$$(1+i)\cdot(2+2i)=$$

Trick for dividing complex numbers :

$$\begin{aligned} \frac{a+ib}{c+id} &= \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id} \\ &= \frac{(ac+bd)+i(bc-ad)}{(c^2+d^2)+i(cd-cd)} \\ &= \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2} \end{aligned}$$

$\frac{3+i}{2-2i} =$

Complex multiplication is easier in polar coordinates: multiple the r's and add the θ's.

$$z = x + iy = r(\cos\theta + i\sin\theta),$$

$$w = u + iv = s(\cos\psi + i\sin\psi),$$

$$\begin{aligned} z \cdot w &= r \cdot s(\cos\theta + i\sin\theta) \cdot (\cos\psi + i\sin\psi) \\ &= (r \cdot s)[\cos\theta\cos\psi - \sin\theta\sin\psi + i\sin\theta\cos\psi + i\cos\theta\sin\psi] \\ &= (r \cdot s)[\cos(\theta + \psi) + i\sin(\theta + \psi)] \end{aligned}$$

Square root of $-x$ is $i\sqrt{x}$.

So negative numbers have square roots, if you allow complex numbers.

Multiply using polar coordinates:

$$(1+i)(-1+i)$$

Quadratic formula for $ax^2 + bx + 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

always works if you allow complex values.

What are roots of $x^2 - x + 4$?

Fundamental theorem of algebra: every polynomial of degree n has n complex roots (some may be repeated).

Complex numbers are essential to applied math (differential equations, Fourier series, financial predictions), enginering (fluid flow, aircraft wings, electric fields), physics (quantum mechanics),...

MAT 342 is all about complex functions.

Complex numbers make some things easier to compute.