MAT 126.01, Prof. Bishop, Tuesday, Nov 17, 2020
Last minute questions on Midterm 3
Section 7.1 Parametric equations

In a usual function the y coordiate is given as a function of x

$$y = f(x).$$

In a parametric equation the x and y coordinates are both given as functions of a third parameter t

$$(x(t), y(t))$$

If $x(t) = t$, the two ideas are the same.

But in general a parametric equation describes curves that are not graphs of functions.

Easiest case is when $x(t) = t$. Then plot of $(x(t), y(t))$ is just graph of $y(t)$.

Eliminating the parameter.

Idea is to write the two equations $x = x(t)$ and $y = y(t)$ as one equation involving x and y.

Example: $x(t) = t^2 - 3$, $y(t) = 2t + 1$.

Example: $x(t) = \cos(t)$, $y(t) = \sin(t)$.

Example: Find equation for $x(t) = 2\cos(t)$, $y(t) = \sin(t)$. What kind of shape is this?

Example: Find equation for $x(t) = \sec(t)$, $y(t) = \tan(t)$. What kind of shape is this?

Find a parametrization of $y = 2x^2 - 3$.

Find a different parametrization of $y = 2x^2 - 3$.

What curve does a point on a rolling wheel follow? Called a cycloid.

Assume radius is a.

Assume wheel takes time 2π to make one rotation (makes equation easier).

Then center moves by $x(t) = at$, $y(t) = a$.

Point on bottom of wheel moves by

$$x(t) = at + a\sin(-t) = at - a\sin t = a(t - \sin t),$$
$$y(t) = a - a\cos(-t) = a(1 - \cos t),$$

A wheel of radius b rolling inside a circle of radius a:

$$x(t) = (a-b)\cos t + b\cos(\frac{a-b}{b}t),$$

$$y(t) = (a-b)\sin t + b\sin(\frac{a-b}{b}t),$$