MAT 126.01, Prof. Bishop, Thursday, Oct 22, 2020
Section 2.8: Exponential Growth and Decay
Quiz 7 review

Definition of exponential growth:

$$y = y_0\ e^{kt}, k > 0.$$

$$y = y_0\ \exp(kt), k > 0.$$

A population of bacteria grows according to

$$f(t) = 200e^{.02t}$$

where t is in minutes. How many bacteria are there after 5 hours?

When are there $100,000$ bacteria?

Evaluate

$$\lim_{n\to\infty}(1+\frac{1}{n})^n.$$

If a bank account has an annual interest rate of r percent paid once a year, and P is the original amount, then after one year the account is worth

$$P(1+r).$$

After t years it is worth

$$P(1+r)^t.$$

Compounded n times in one year gives

$$P(1+\frac{r}{n})^n.$$

Continuous compounding is the limit compounding period $\to 0$.

$$\lim_{n\to\infty} P(1+\frac{r}{n})^n = Pe^r.$$

If we continuously compound for t years the value is

$$Pe^{rt}.$$

Assume the fish population in a pond grows exponentially. Suppose it starts at 500 and after 6 months is a 1000. How long before the population hits 10,000?

Exponential decay:

$$y = y_0 \, e^{-kx}, k > 0.$$

Carbon 14 is a material that accumulates in living things, and stops accumulating when they die. The amount of Carbon 14 then starts to decay exponentially as it changes to Nitrogen-14.

The half-life of Carbon-14 is 5730. Find k in the decay equation

$$y = y_0\, e^{-kt}.$$

Some pottery contains only 70% of its original Carbon-14. How old is it?

Newton's law of cooling: If an object with initial temperature T_0 is placed in an environment with constant temperature T_a (a for ambient) its temperature at time t

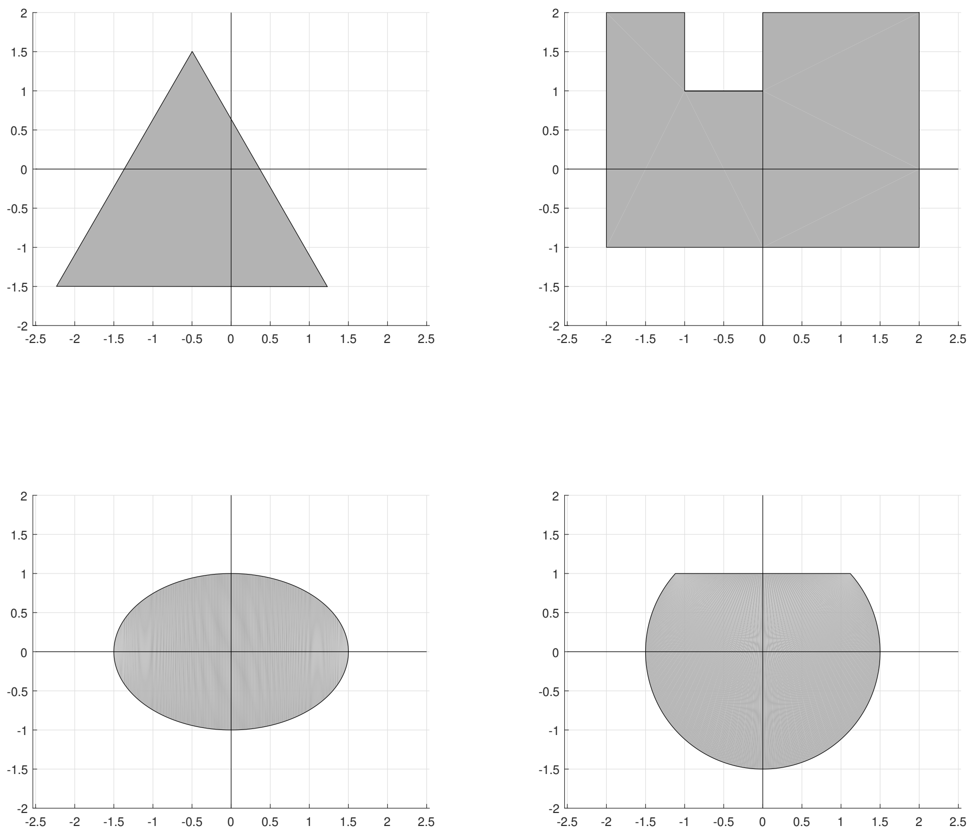
$$T = (T_0 - T_a)e^{-kt} + T_a.$$

This says the object goes to the ambient temperature exponentially fast.

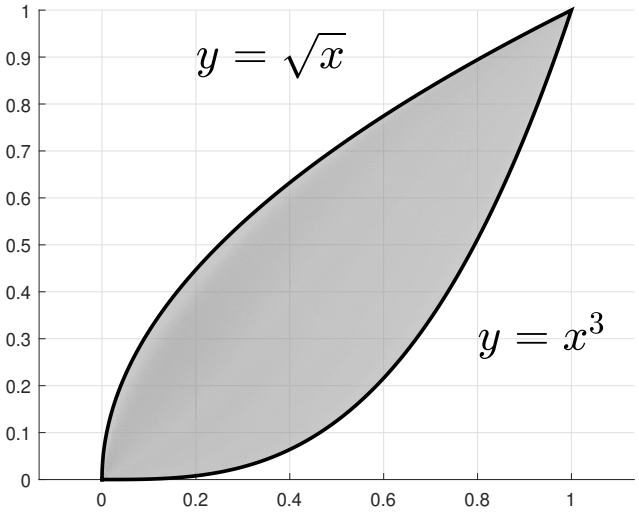
Quiz 7 review:

- 2 questions: finding center of mass by eye
- 3 questions: given region find mass, x-moment, y-moment
- 1 question: theorem of Pappus
- 2 questions: derivative of exponentials
- 2 questions: Newton's law of cooling

For each figure say where center of mass is: the origin, or what quadrant, or which axis (e.g., positive x-axis, negative y-axis,...).



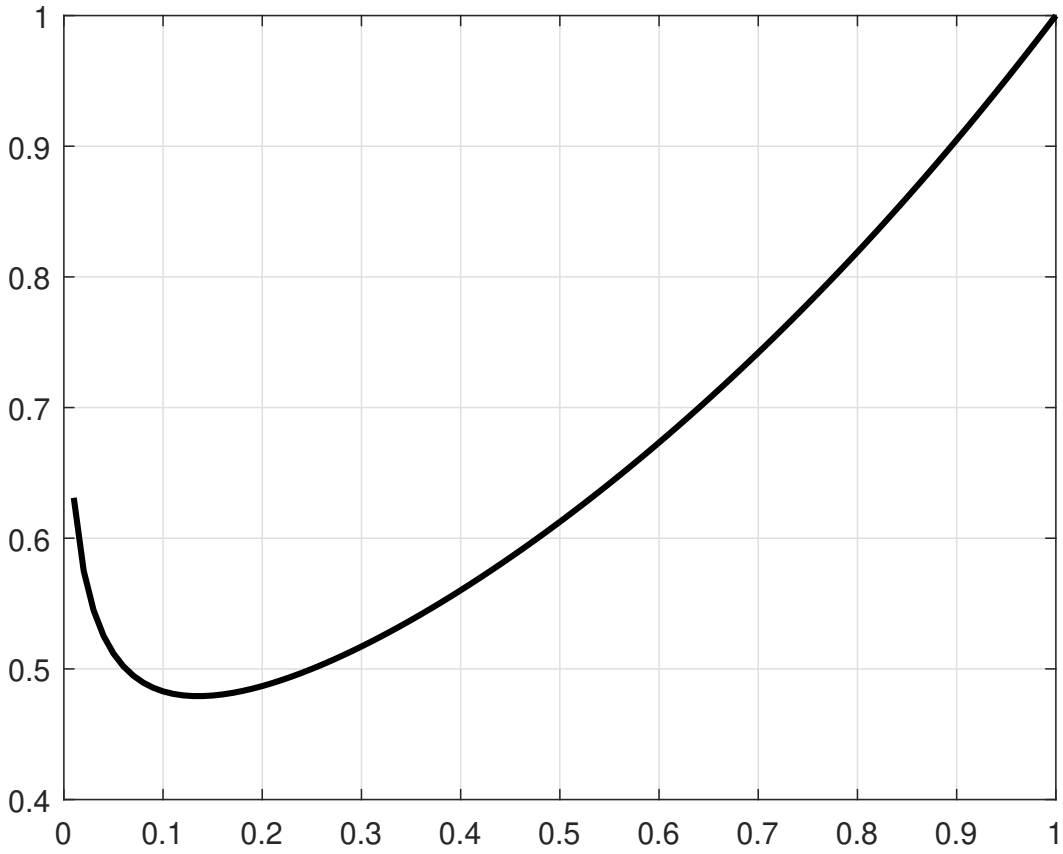
Compute the mass, x-moment and y-moment for the following region.



An equilateral triangle with base $[1, 2]$ on the x-axis is rotated around the y-axis. Use the Theorem of Pappus to compute the volume of this region.

Compute the derivative of $f(x) = x^{\sqrt{x}}$.

What is minimal value of f on $(0, 1)$?


1
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0.4
0.5
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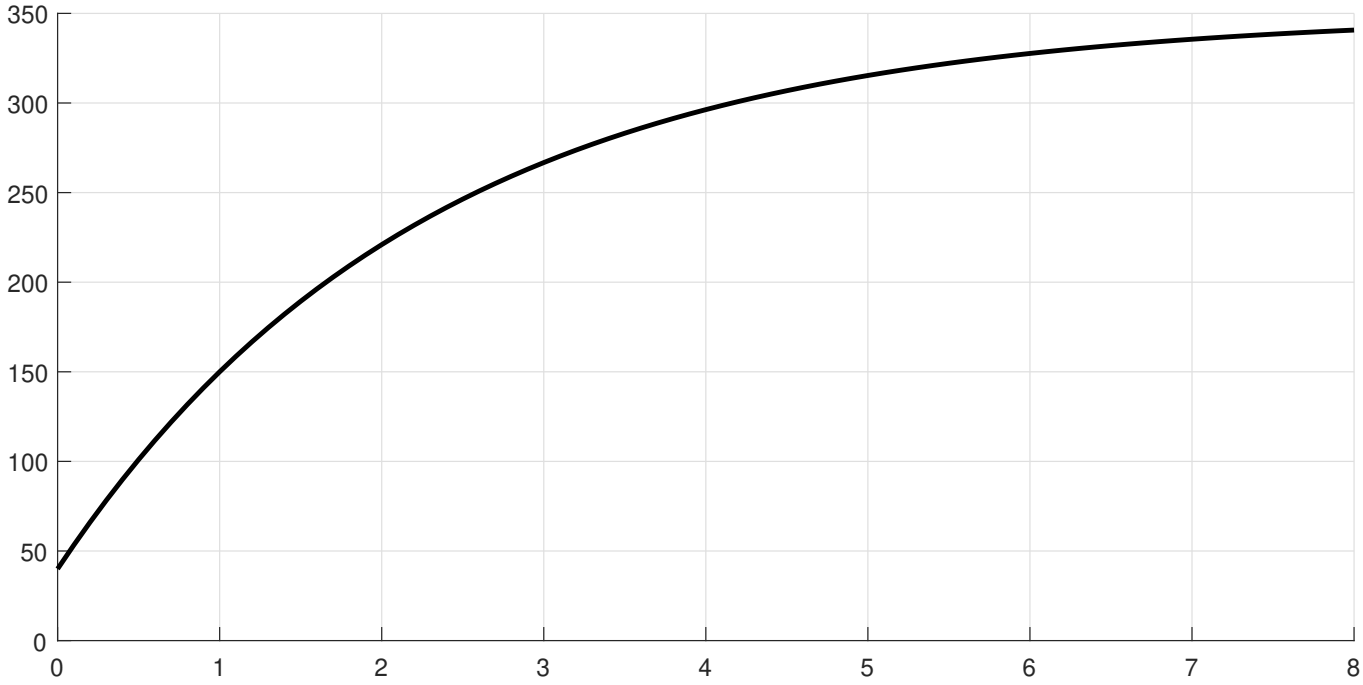
0.8
0.9
1

A 40° degree turkey is put into a 350° degree oven. What is the temperature as a function of time, according to Newton's law of cooling? Leave k as a parameter.

After an hour, the turkey is at 100°. What is the value of k.

When does the turkey reach 330°?


350
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