MAT 126.01, Prof. Bishop, Thursday, Sept. 10, 2020

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Section 1.5, Substitution

- Recall the chain rule for derivatives.
- Substitution rule for integrals (indefinite integrals).
- Substitution rule for integrals (definite integrals).
- Polynomial examples
- Trigonometric examples
    - Review common trig formulas
    - Using trig identities to simplify integrals.

$$f(g(x)) = \int \big(f(g(x))\big)' = \int \underbrace{f'(g(x)) \cdot g'(x)}_{\text{chain rule}}$$

Substitution Rule:

Suppose f, g, g' are continuous. The

$$\int \boxed{f(g(x))}\,\boxed{g'(x)dx} = \int (f(g(x))'dx = f(g(x)) + C.$$

$$\int \sin^2(x)\cos(x)dx = \int [\sin(x)]^2 \cos(x)dx$$

Example: Let $u = \sin(x)$, so $\frac{du}{dx} = \cos(x)$ or $du = \cos(x)dx$.

$$\int \sin^2(x)\cos(x)dx = \int [u]^2 du = \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}\sin^3(x) + C$$

$$\left(= \frac{\cancel{1}}{\cancel{3}}\,\cancel{3}\sin^2(x)\cos x + 0\right) ✓$$

Sometimes we need to multiply and divide by a factor to get du correct.

Find $\int \sqrt{3x+2}dx$

$u = 3x+2 \rightarrow \frac{du}{dx} = 3 \rightarrow du = \underbrace{3dx}$

$= \frac{1}{3} \int \sqrt{3x+2} \cdot \underbrace{3dx} = \frac{1}{3} \int \sqrt{u}\ \underbrace{du}$

$\int x^n = \frac{1}{n+1} x^{n+1}$

$n = \frac{1}{2} \quad n+1 = \frac{3}{2}$

$= \frac{1}{3} \int u^{1/2} du$

$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9}(3x+2)^{3/2} + C$

Find $\int x \sin(x^2) dx$

$\boxed{u = x^2} \quad \frac{du}{dx} = 2x \quad \boxed{du = 2x\,dx}$

$\frac{1}{2} \int \boxed{\sin(x^2)} \boxed{2x\,dx} = \frac{1}{2} \int \boxed{\sin(u)} \boxed{du}$

$= -\frac{1}{2}\cos(u) + C = -\frac{1}{2}\cos(x^2) + C$

$(e^x)' = e^x$

$\int e^x = e^x + c$

Find $\int e^{4x}dx$

$\boxed{u = 4x}$ $\quad \frac{du}{dx} = 4 \quad \boxed{du = 4\,dx}$

$$\int e^{4x}dx = \int e^{4x}\frac{4}{4}dx = \frac{1}{4}\int e^{4x}\,4\,dx$$

$$= \frac{1}{4}\int e^u du = \frac{1}{4}e^u + c = \frac{1}{4}e^{4x} + C$$

Find $\int xe^{x^2}dx$

$u = x^2 \quad \frac{du}{dx} = 2x \quad du = 2x\,dx$

$$= \frac{1}{2}\int e^{x^2}2x\,dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \boxed{\frac{1}{2}e^{x^2} + c}$$

Find $\int (x+1)\cos(x^2+2x+1)dx$

$u = x^2 + 2x + 1 \qquad du = (2x+2)\,dx$

$\qquad\qquad\qquad\qquad\qquad = 2(x+1)\,dx$

$= \frac{1}{2}\int \cos(x^2+2x+1)\,2(x+1)\,dx = \frac{1}{2}\int \cos u \, du$

$= \frac{1}{2}\sin u + C = \frac{1}{2}\sin(x^2+2x+1) + C$

Find $\int \sin^{10}(x)\cos(x)dx \qquad u = \sin x \quad du = \cos x\,dx$

$\int u^{10}\,du = \frac{1}{11}u^{11} + C$

$= \frac{1}{11}\sin^{11}(x) + C$

Using substitution with definite integrals is a little trickier. You you also have to change the limits of integration:

Find $\int_0^1 x\sin(\underbrace{\pi x^2}_{u})dx$. $u = \pi x^2$ $\boxed{du = \underbrace{2\pi x}\, dx}$

$$= \frac{1}{2\pi}\int_0^1 \sin(\pi x^2)\, \boxed{2\pi\, x\, dx} = \frac{1}{2\pi}\int_0^{\pi} \sin(u)\, \boxed{du}$$

$x=0$ $u = \pi x^2 = 0$

$x=1$ $u = \pi 1^2 = \pi$

$$= \frac{1}{2\pi}\left(-\cos u\right)\Big|_0^{\pi}$$

$$= \frac{1}{2\pi}\left(-(-1)-(-1)\right)$$

$$= \frac{1}{2\pi}(1+1) = \frac{2}{2\pi} = \frac{1}{\pi}$$

Find $\int_1^2 \frac{2\ln(x^2+1)\cancel{x}}{x^2+1}dx$.

$u = \ln(x^2+1)$ $du = \frac{2x}{x^2+1}$

$$= \int u\, du = \frac{1}{2}u^2 = \frac{1}{2}\left(\ln(x^2+1)\right)^2\Big|_1^2$$

$$= \frac{1}{2}\left[(\ln 5)^2 - (\ln 2)^2\right]$$

Find $\int_2^4 \frac{x}{\sqrt{x^2-1}}dx$.

$u = x^2 - 1 \qquad \frac{du}{dx} = 2x \qquad du = 2x\,dx$

$$\frac{1}{2}\int_2^4 (x^2-1)^{-\frac{1}{2}}\, 2x\, dx = \frac{1}{2}\int u^{-\frac{1}{2}}\, du$$

$$= \frac{1}{2}\cdot\frac{1}{1/2}\, u^{1/2}$$

$$= 1\cdot (x^2-1)^{1/2}\Big|_2^4$$

$$= \sqrt{15} - \sqrt{3}$$

$$\int u^n = \frac{1}{n+1}\, u^{n+1}$$

~~Find~~ $\cancel{\int_1^2 \frac{2\ln(x^2+1)}{x^2+1}dx}$.

Sometimes some algebra or trig identites are helpful:

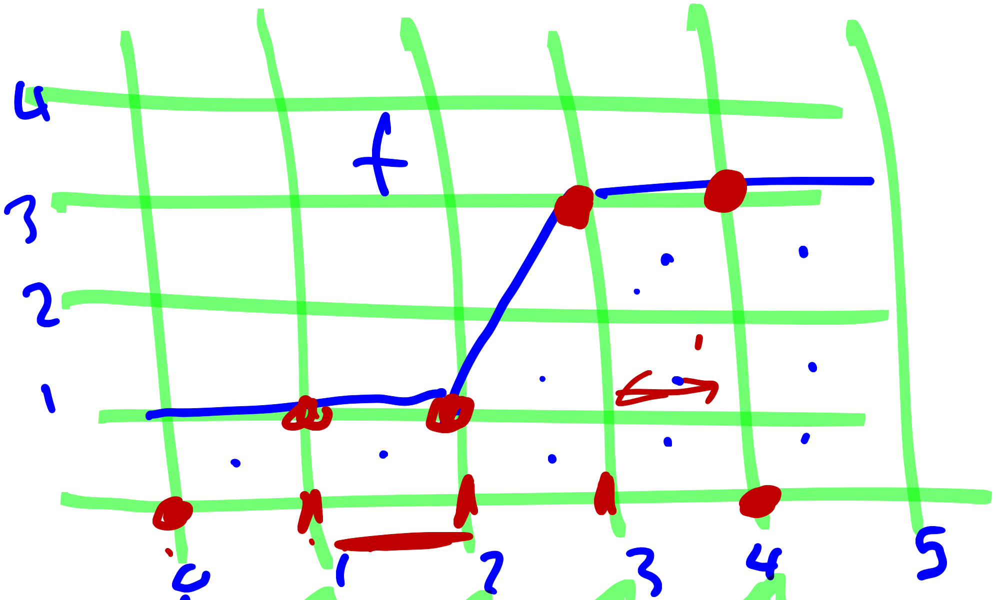
Find $\int \cos^3(x) dx$.

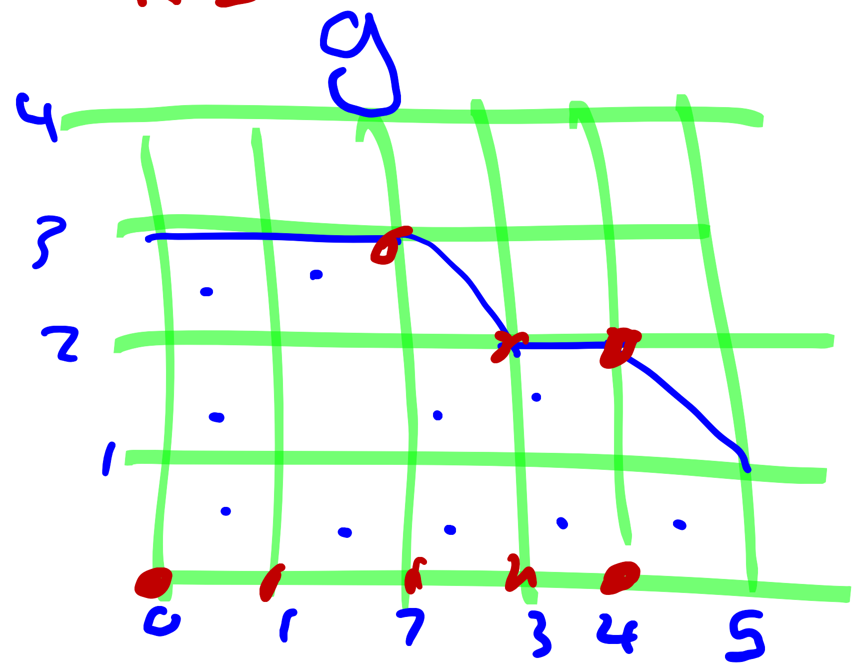
I will do this next Tuesday
Sept 15.

Sometimes some algebra or trig identites are helpful:

Find $\int_0^\pi \sin^2(x)dx$.

OFFICE HOURS





hard

Approx $\int_0^4 f(x)g(x)\,dx$ 4 ints, right endpts

$$\approx 1\cdot f(1)g(1) + 1\cdot f(2)g(2) + 1\cdot f(3)g(3) + 1\cdot f(4)\cdot g(4)$$

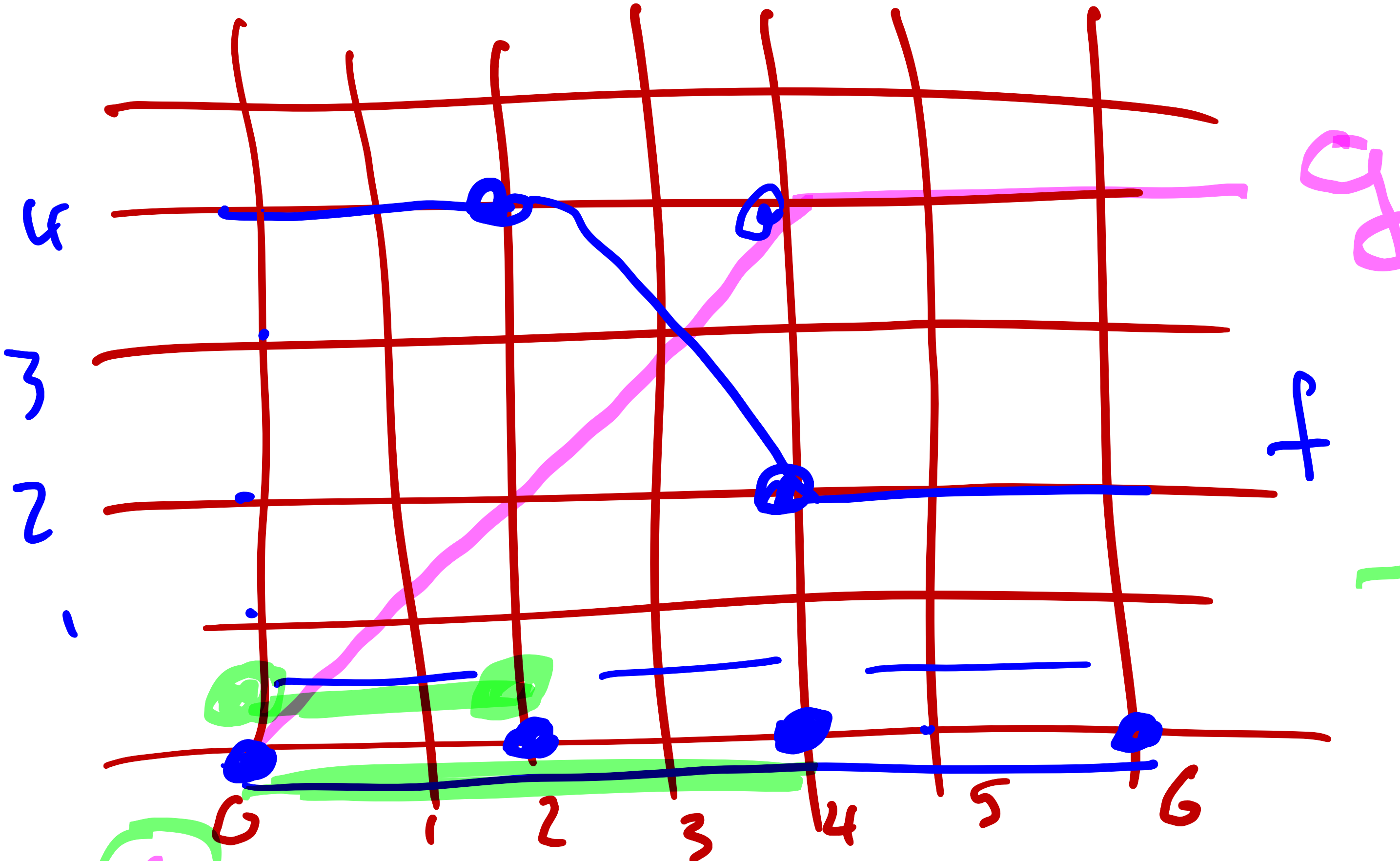
$$= 1\cdot 3 + 1\cdot 3 + 3\cdot 2 + 3\cdot 2$$

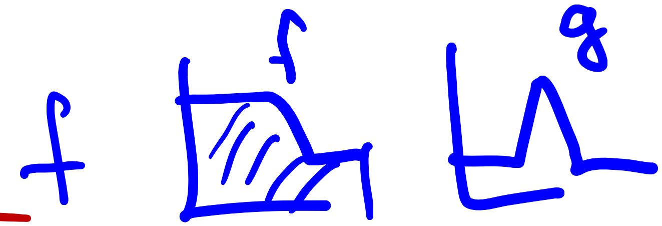
$$= 18$$

$\int_0^5 f+g = \int f + \int g$

easy

$10 + 12$
$= 22$





$\int_0^6 f(g(x))\,dx$ = Estimate using left endpts, 3 intervals.

$$\approx 2 \cdot f(g(0)) + 2f(g(2)) + 2f(g(4))$$

$$= 2f(0) + 2f(2) + 2f(4)$$

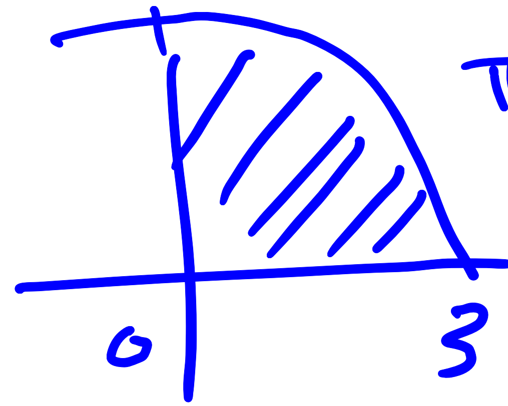
$$= 8 + 8 + 4 = 20$$

$$\frac{3}{n}\sum_{k=0}^{n-1}\left(9-\left(\frac{3k}{n}\right)^2\right)^{1/2}$$

x

Δx

$$\int_0^3 (9-x^2)^{1/2}\,dx$$

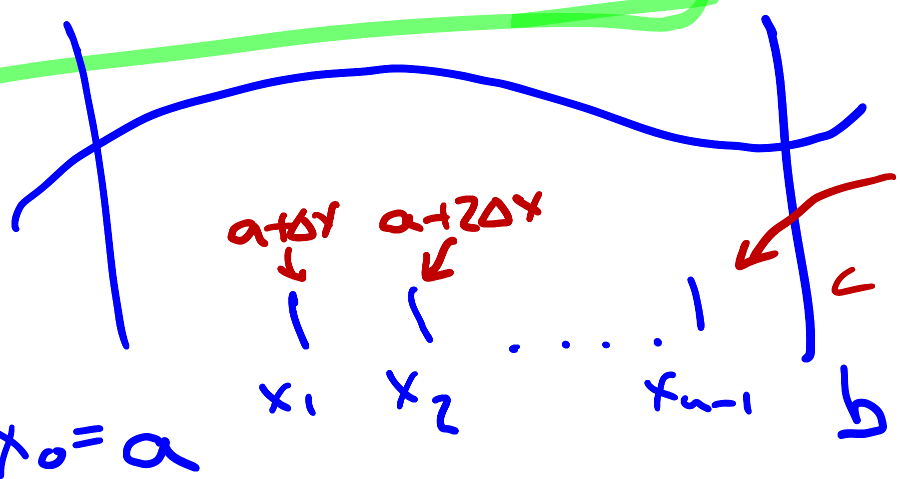


πr^2

$\frac{\pi \cdot 9}{4}$

$a=0$

$$\boxed{\Delta x\sum_{k=1}^{n} f(a+k\Delta x)\cdot \cancel{\Delta x}}$$



$$\int_3^0 \sqrt{x^2}$$

$x = 9-\left(\frac{3k}{n}\right)^2$

1 2 3

1 4 9

$$\frac{1}{2}\sum_{k=1}^{n}\Big(\underbrace{\frac{8k}{n}+2}_{x}\Big)^2 \underbrace{\frac{4\cdot 2}{n}}_{\Delta x}$$

$\Delta x = \frac{4}{n}$

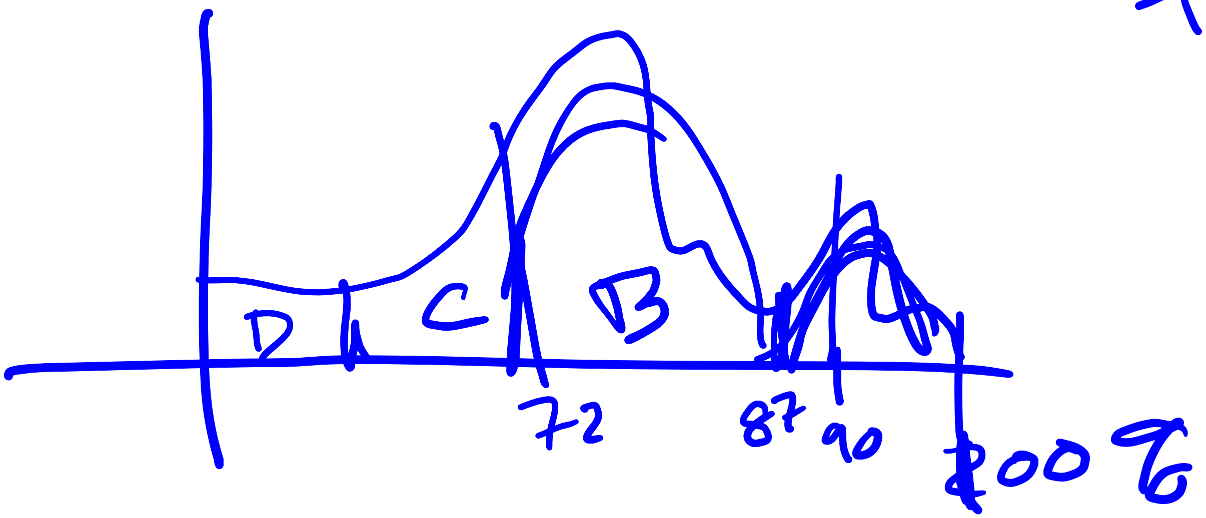
$$\int_2^6 x^2\,dx \checkmark$$

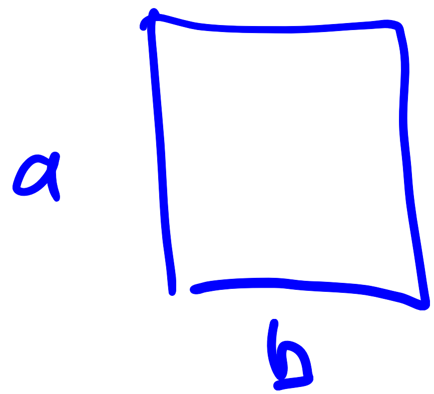
or

$$\boxed{\int_0^4 (x+2)^2\,dx}$$

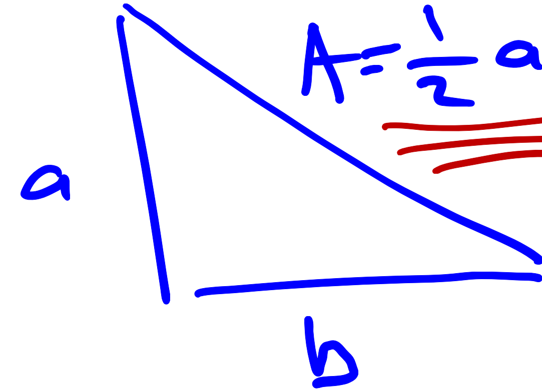
$u = x+2$

$$\int_2^6 u^2$$

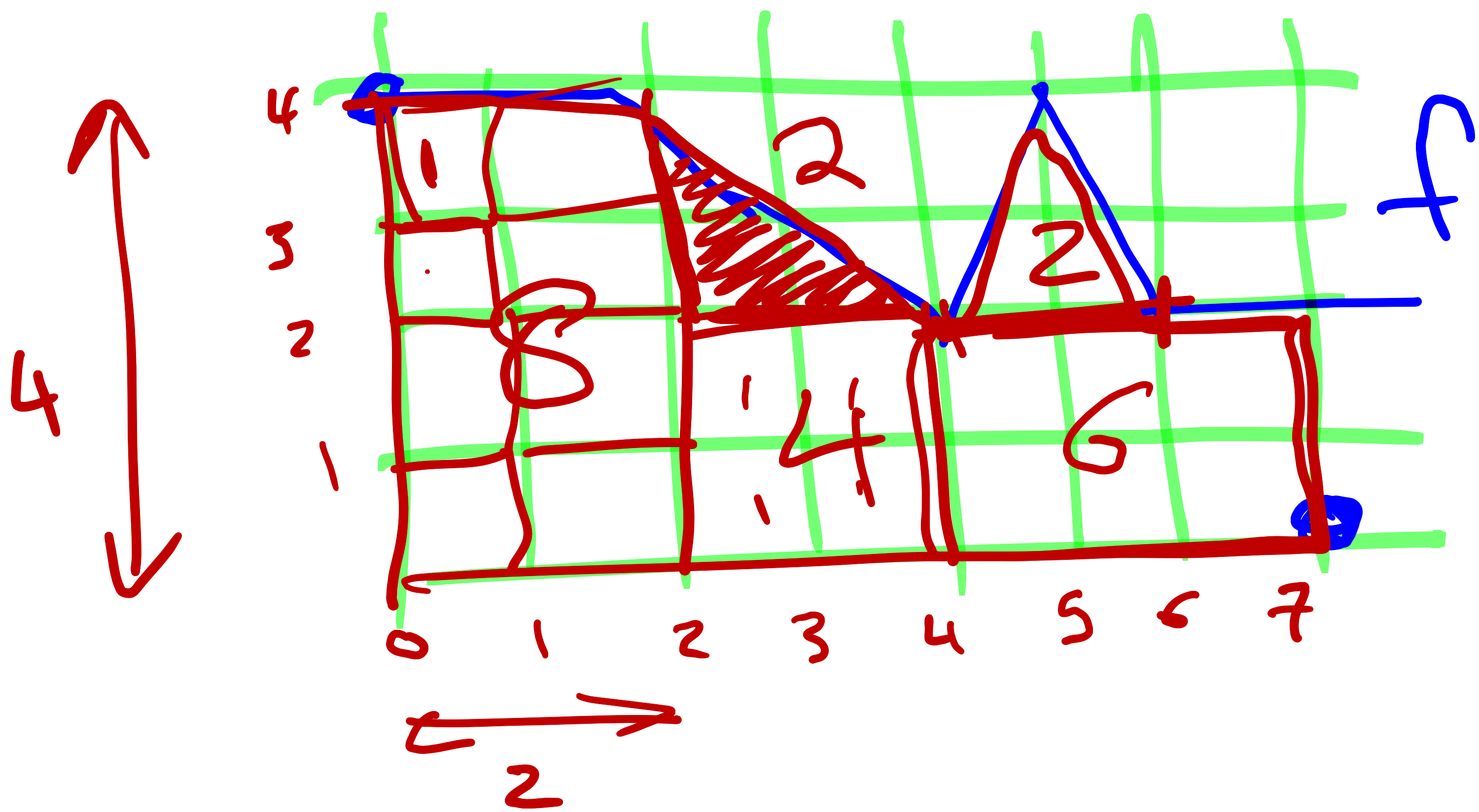




$A = a \cdot b$



$A = \frac{1}{2}ab$



$\int_0^7 f\,dx$
$= 22$