

**MAT 126.01, Prof. Bishop, Tuesday, Sept. 15, 2020**

**Tuesday, September 15, 2020**  
**Finish Section 1.5, Substitution**  
**Section 1.6, Substitution**

- ▶ Using trig substitutions with substitution
- ▶ Definition of natural logarithm
- ▶ Other logarithms
- ▶ Differentiation of logarithms
- ▶ Definition of natural exponents
- ▶ Other bases
- ▶ Differentiation of exponentials
- ▶ Examples

Sometimes some algebra or trig identities are helpful:

Find  $\int \cos^3(x) dx$ .

Sometimes some algebra or trig identities are helpful:

Find  $\int_0^\pi \sin^2(x) dx$ .

We define

$$\ln(x) = \int_1^x \frac{1}{t} dt.$$

Then  $\frac{d}{dx} \ln x = \frac{1}{x}$  is obvious.

Derive  $\ln(1) = 0$

Derive  $\ln\left(\frac{1}{x}\right) = -\ln(x)$



Derive  $\ln(xy) = \ln(x) + \ln(y)$

Derive  $\ln(x^p) = p \ln(x)$

Define  $\log_b = \ln(x) / \ln(b)$ .

$$\frac{d}{dx} \log_b x = \frac{1}{x \ln b} .$$

Derive  $\int \ln x dx = x \ln x - x + C = x(\ln x - 1) + C$

Derive  $\int \log_a x dx = \frac{x}{\log a} (\ln x - 1) + C$

Define  $e^x$  as the inverse of  $\ln x$ .

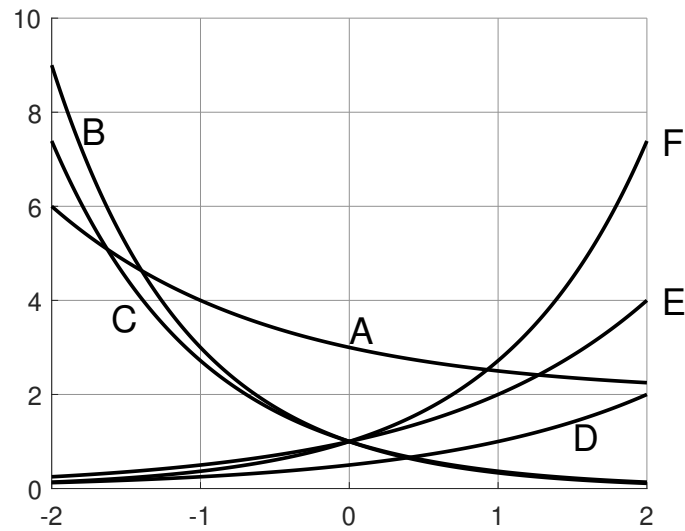
Derive  $\frac{d}{dx}e^x = e^x$  .

Define  $a^x = e^{x \ln a}$

Derive  $\frac{d}{dx}a^x = a^x \ln a$  .

Find the graph of  $e^x$ .

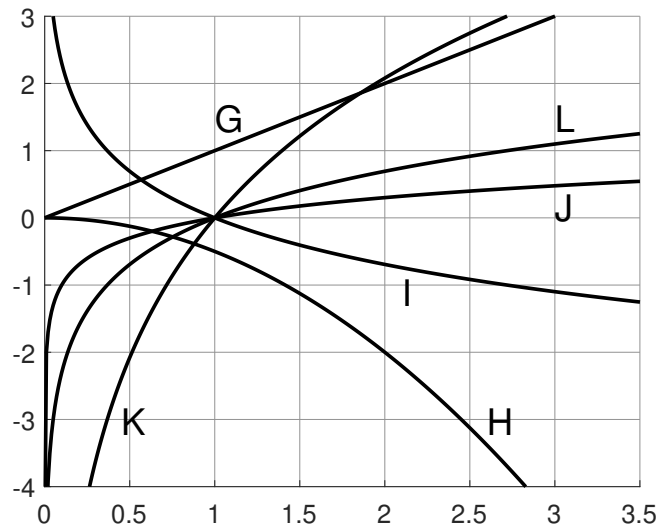
Find the graph of  $3^{-x}$ .





Find the graph of  $\ln x$ .

Find the graph of  $\ln \frac{1}{x}$ .



Find  $\int \frac{2x+3}{x^2+3x+4} dx$

Find  $\int_0^{\pi/2} \frac{\sin x}{\cos x + 1} dx$

Find  $\int e^x \sqrt{1 + e^x} dx$

Find  $\int \frac{1}{x \ln x} dx$































