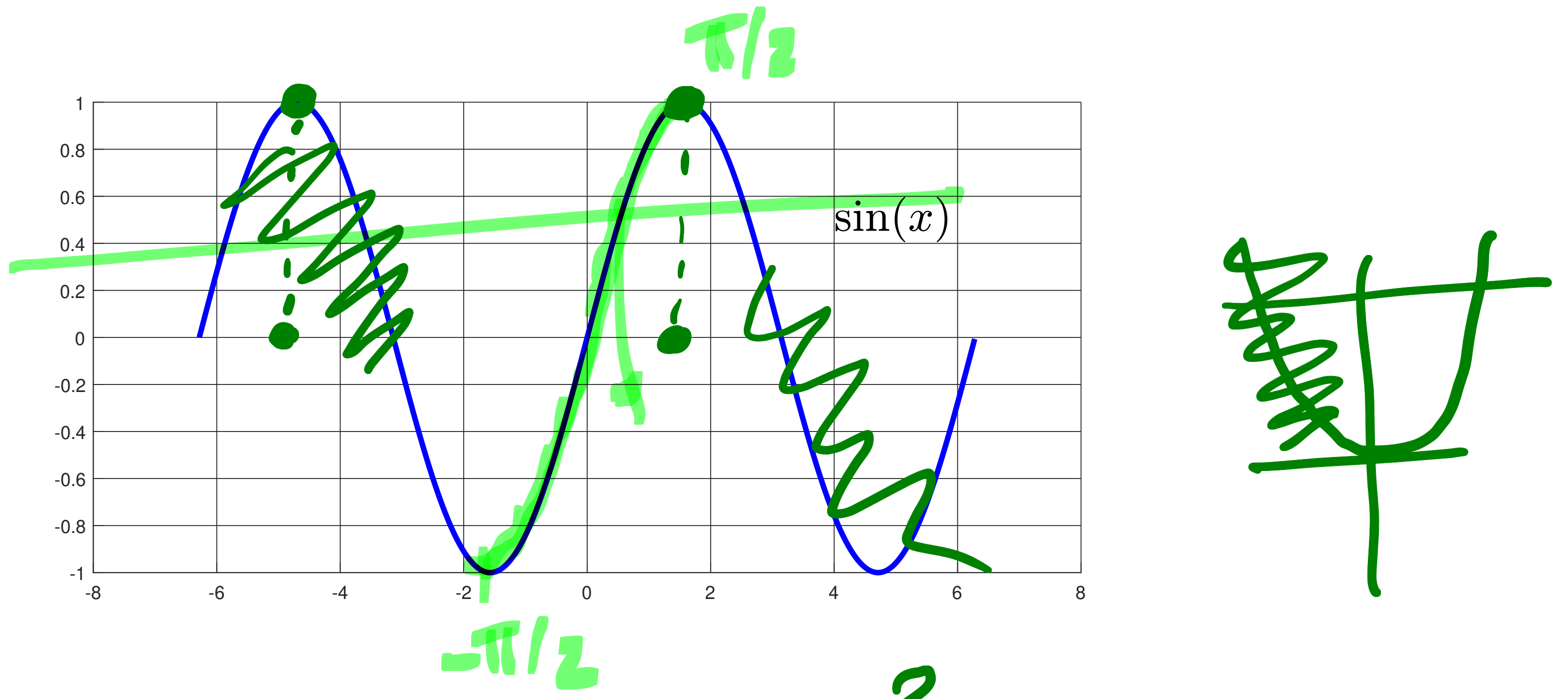
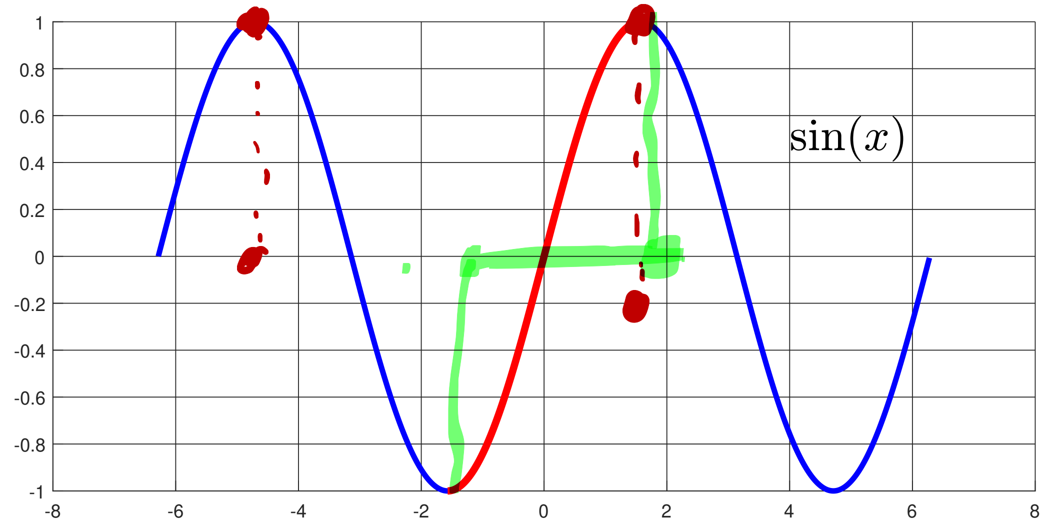
MAT 126.01, Prof. Bishop, Thursday, Sept. 17, 2020

Thursday, September 17, 2020
Section 1.7, Integrals resulting in Inverse Trig Functions.

- Derivatives of inverse functions
- Sin and its inverse
- Tan and its inverse
- Sec and its inverse
- Examples

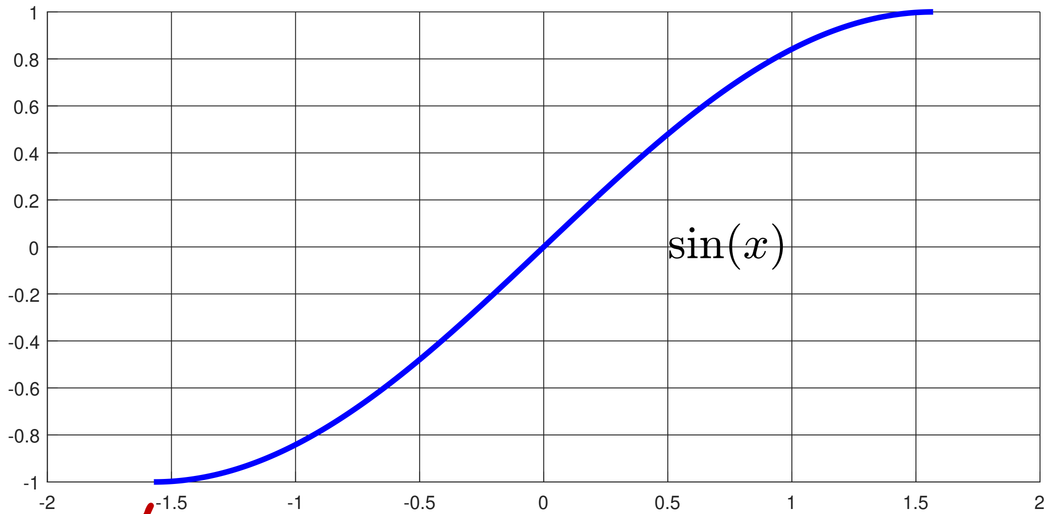


arcsin x $\sin^{-1} x$?

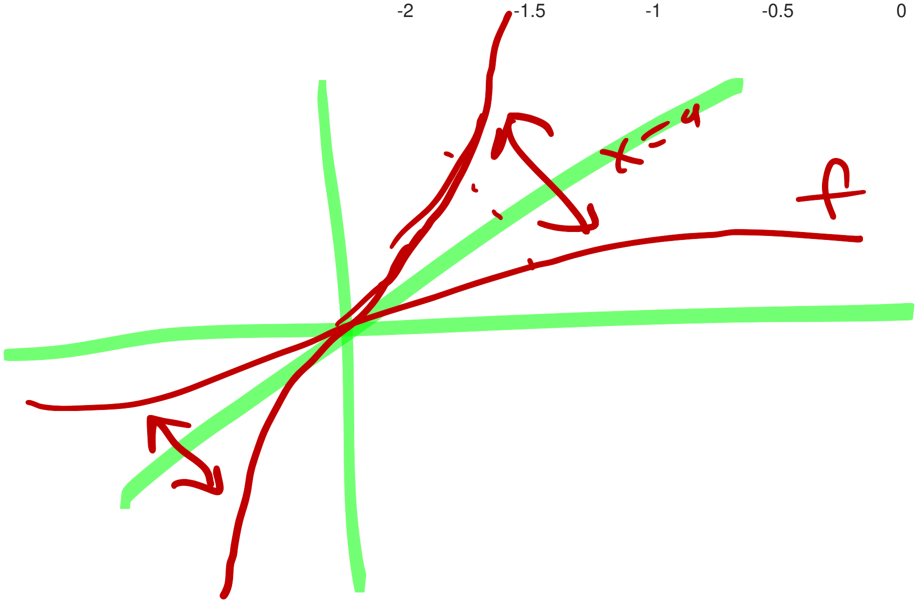


$\sin^{-1}(x)$ = inverse of sin on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

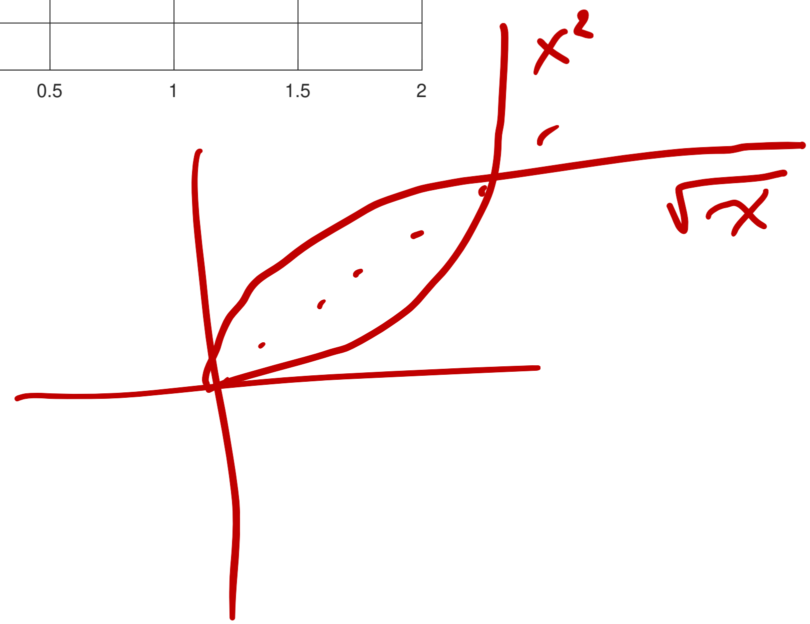
$\sin^{-1}(1)$



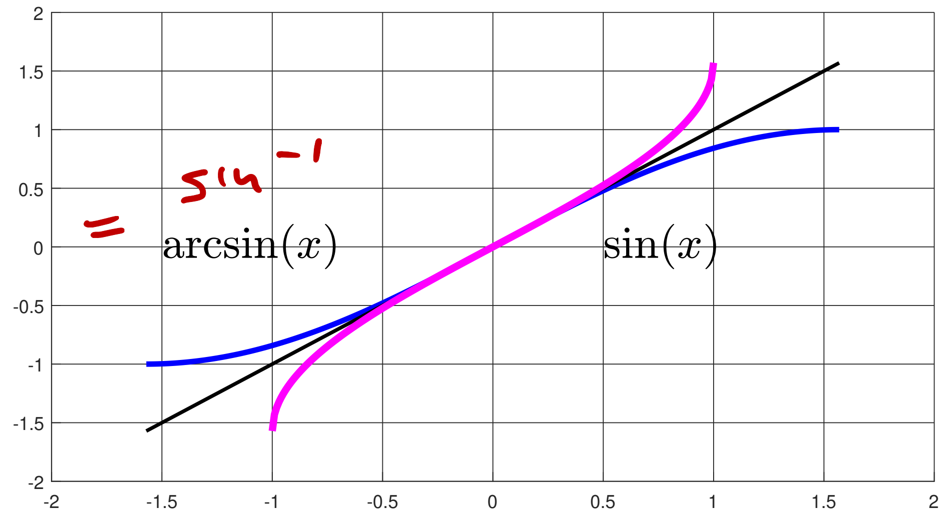
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$\sin(x)$



x^{-4}
f



x^2
$\sqrt{x}$



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$= \sin^{-1}$
$\arcsin(x)$
$\sin(x)$

Functions f and g are inverses if $f(g(x)) = x$.

Examples e^x and $\ln x$

Examples x^2 (for $x > 0$) and $\sqrt{x}$

A function must be 1-to-1 to have an inverse

Many common functions have to restricted to have an inverse. Like x^2.

Graph of inverse is reflection of graph of f over diagonal $y = x$

$f(g(x)) = 1$

If f and g are inverse functions then

$$g'(x) = \frac{1}{f'(g(x))}$$

Derive using chain rule.

Apply to $\sin(x/a)$ and $a\sin^{-1}(x)$.

Derive $\boxed{\frac{d}{dx}\sin^{-1}(x/a) = \frac{1}{\sqrt{a^2-x^2}}}$

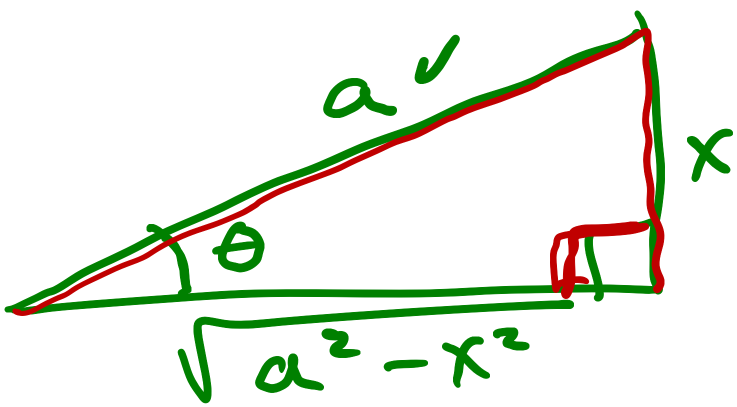
$$f(x) = \sin\frac{x}{a}$$

$$f'(x) = \frac{1}{a}\cos\frac{x}{a}$$

$$g(x) = \sin^{-1}\left(\frac{x}{a}\right)$$

$$g'(x) = \frac{1}{f'(g(x))}$$

$$\left(\sin^{-1}\left(\frac{x}{a}\right)\right)' = \frac{1}{\frac{1}{a}\cos\Big(\underbrace{\sin^{-1}\frac{x}{a}}_{\theta}\Big)} = \frac{1}{\cancel{\frac{1}{a}}\,\frac{\sqrt{a^2-x^2}}{\cancel{a}}}.$$



$$\theta = \sin^{-1}\frac{x}{a}$$

$$\sin\theta = \frac{x}{a} = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{a^2-x^2}}{a}$$

Thus

$$\boxed{\int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1}(x/a) + C}$$

✓

Q

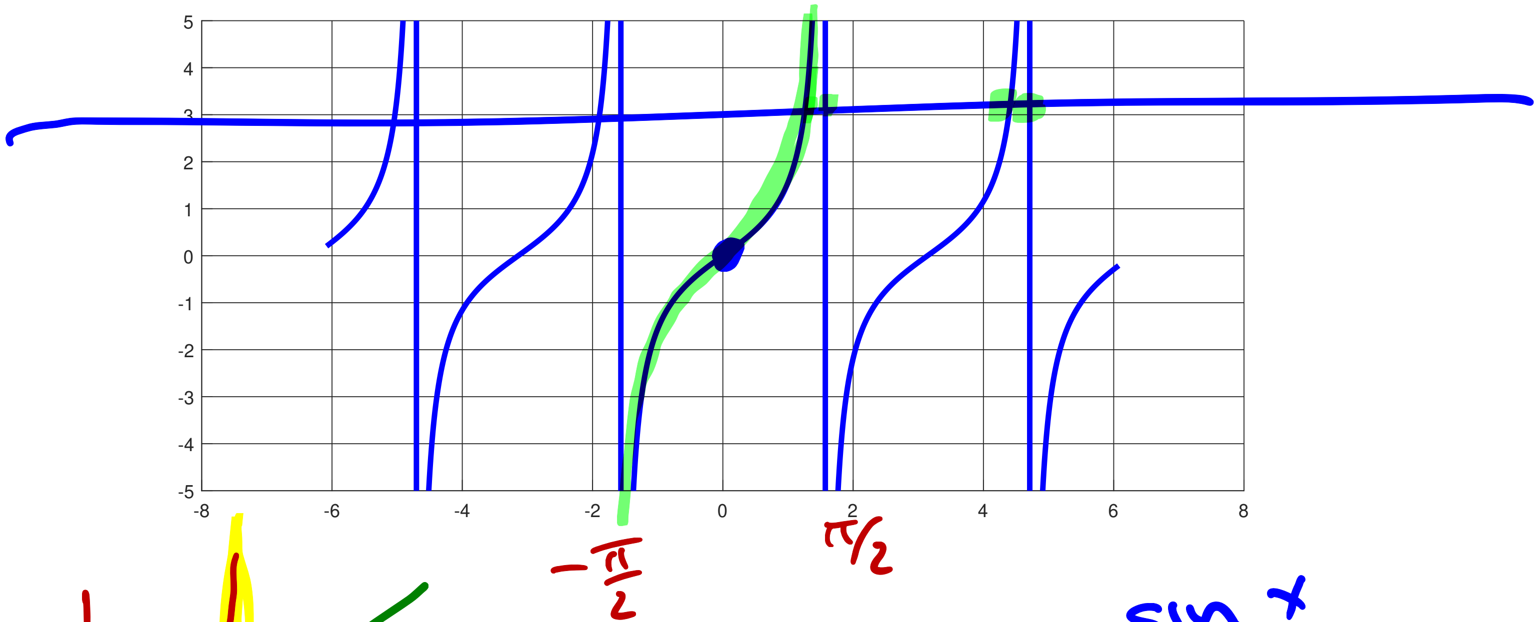
Find $\int_0^3 \frac{1}{\sqrt{9-x^2}}dx$

$$= \sin^{-1}\left(\frac{x}{3}\right)\Big|_0^3$$

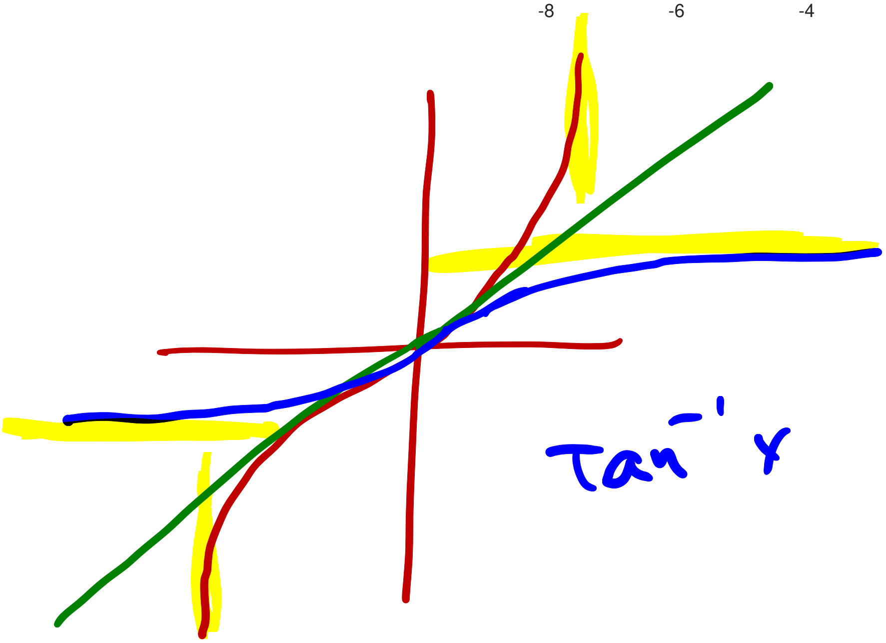
$a = 3 \quad a^2 = 9$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

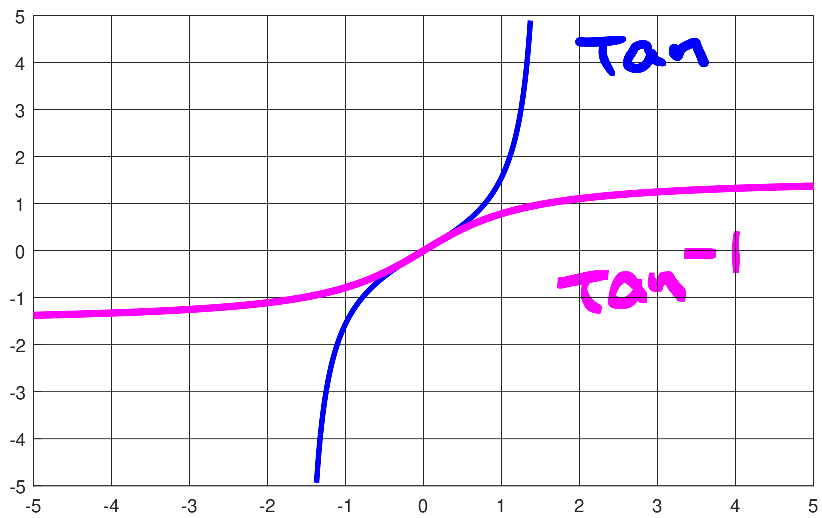
$$= \frac{\pi}{2} - 0$$



$$\tan x = \frac{\sin x}{\cos x}$$



$\tan^{-1} x$



tan
tan⁻¹

Derive

$$\frac{d}{dx}\tan^{-1}(x/a) = \frac{a}{a^2+x^2}$$

$$\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}(x/a) + C$$

Q

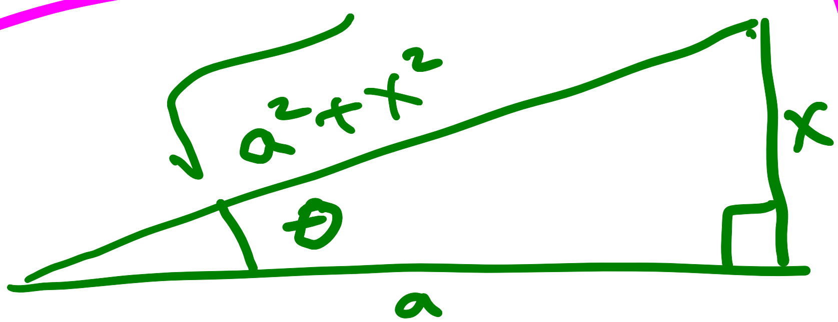
Q

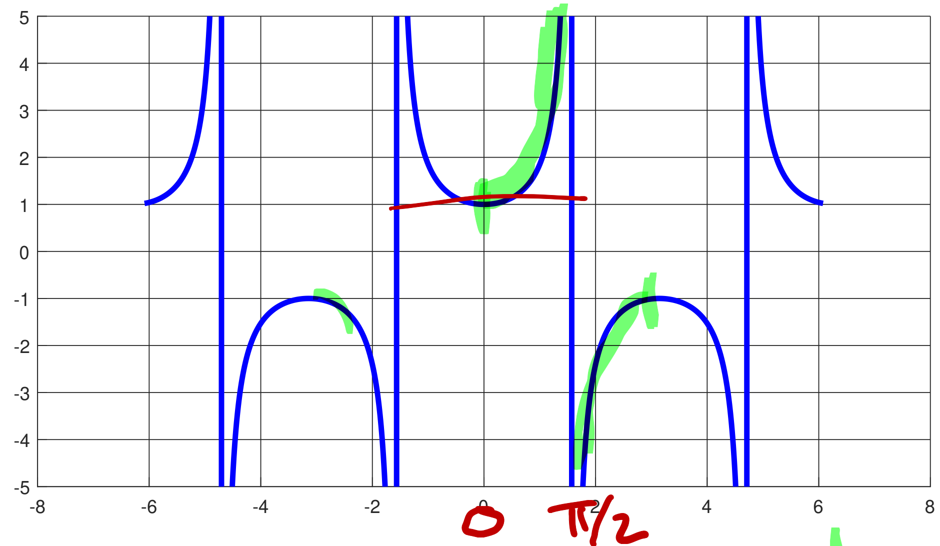
$$\frac{1}{a}\tan(\underbrace{\tan^{-1}x/a}_{\theta}) = x$$

$\theta = \tan^{-1}\frac{x}{a}$

$\tan = \frac{opp}{adj}$

$\tan\theta = \frac{x}{a}$

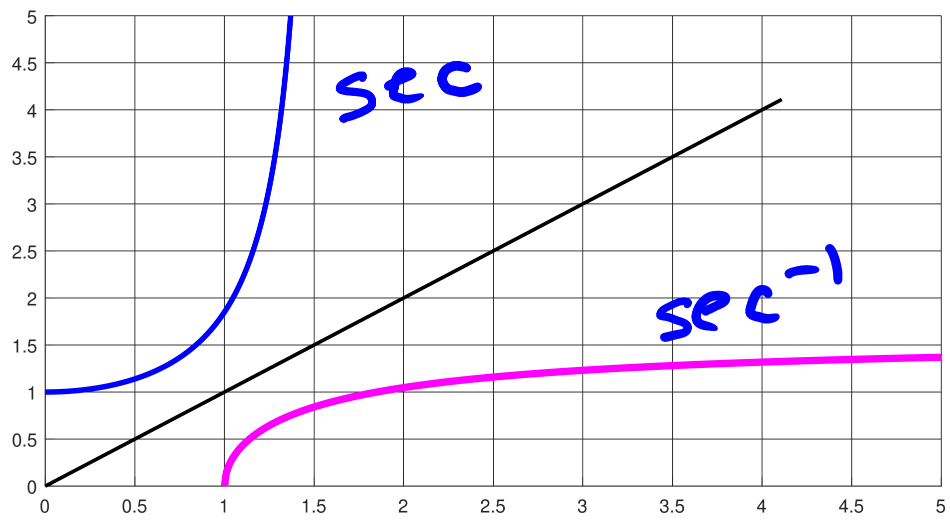




$> 1 \leftarrow$

< -1

$\sec = \frac{1}{\cos}$


5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
sec
sec^{-1}

Derive

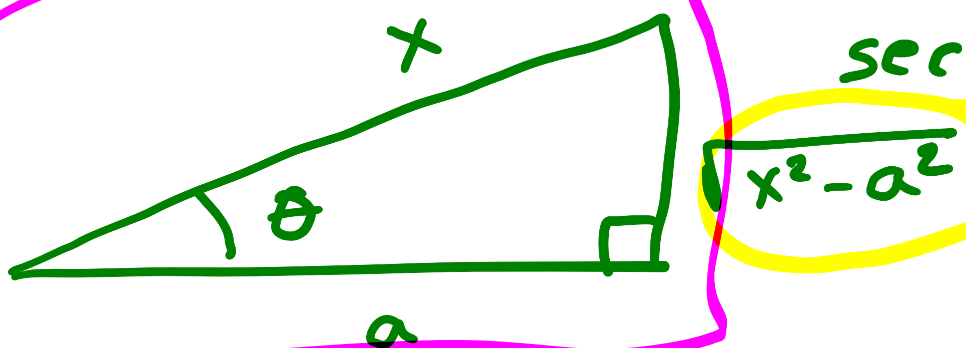
Q

$$\frac{d}{dx}\sec^{-1}(x/a) = \frac{a}{x\sqrt{x^2 - a^2}}$$

$$\int \frac{a}{x\sqrt{a^2 - x^2}} = \frac{1}{a}\sec^{-1}(x/a) + C$$

$\theta = \sec^{-1}\left(\frac{x}{a}\right)$

$\sec\theta = \frac{x}{a} = \frac{\text{hyp}}{\text{adj}}$



Find $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

$$\stackrel{?}{=} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$a = 1$

$$= \sin^{-1}(x) \Big|_0^1$$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \frac{\pi}{2} - 0 = \pi/2$$

Find $\displaystyle\int \frac{dx}{\sqrt{4-9x^2}}$. $= \displaystyle\int \frac{1}{3}\frac{1}{\sqrt{(2/3)^2 - x^2}}\,dx = \frac{1}{3}\sin^{-1}\left(\frac{x}{2/3}\right)$

$= \frac{1}{3}\sin^{-1}\left(\frac{3}{2}x\right)$

? $\sqrt{a^2 - x^2}$

$\sqrt{4-9x^2} = \sqrt{9\cdot\frac{4}{9} - 9x^2}$

$= 3\sqrt{(4/9) - x^2}$

$= 3\sqrt{(2/3)^2 - x^2}$

$\boxed{a = 2/3}$

Find $\int \frac{dx}{\sqrt{x^2+9}}$.

Tan

Find $\int_0^2 \frac{dx}{\sqrt{x^2+4}}$.

tan

Office Hours Begin

$\approx$ 11:15

1st page of Midterm 1

30 = 3 × 10 = 3 Quizzes

$$\sum_{k=0}^{3} 3^k = 3^0 + 3^1 + 3^2 + 3^3$$
$$= 1 + 3 + 9 + 27$$
$$= 40$$

$$\sum_{k=1}^{15} k = 1 + 2 + \boxed{3 + \cdots + 15}$$

3

You may use $\sum_{1}^{n} k = \frac{n(n+1)}{2}$

$$\sum_{k=3}^{15} k$$

4

(f) 120 ✓

$$= \frac{15 \cdot 16}{2} = 15 \cdot 8$$

$$\begin{array}{r} 15 \\ \times 8 \\ \hline 120 \end{array}$$

$$\sum_{k=0}^{n} a_k \left(\frac{k}{n}\right) \quad \text{"x"}$$

$k=0 \qquad k=n$

$$1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{101} =$$

$$\sum_{k=1}^{100} \frac{1}{k} \to 1 + \frac{1}{2} + \quad \times$$

$$\sum_{k=1}^{50} \frac{1}{2k+1} = \frac{1}{3} + \frac{1}{5} + \cdots \quad \times$$

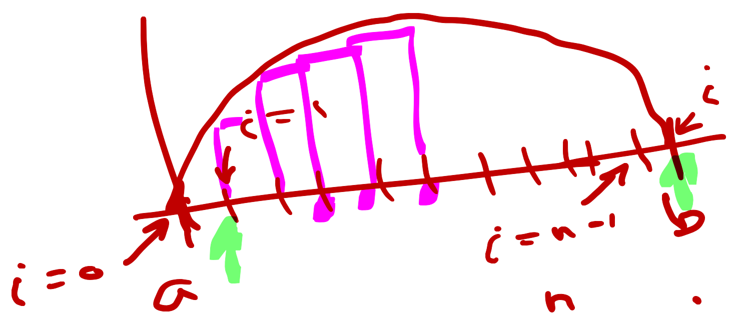
$$\sum_{k=0}^{50} \frac{1}{2k+1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{101} \quad \checkmark$$

$\vdots$

Write Approx for $\int_0^{\pi} \sin(x)\,dx$

6.4 using $\boxed{10}$ intervals Right pts. $\qquad i, k$

n

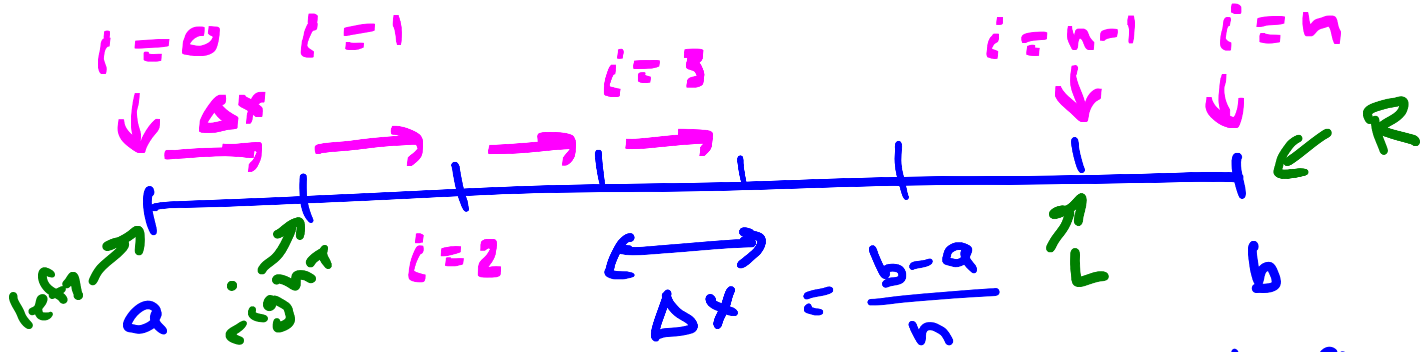


$$\Delta x = \frac{\pi}{n}$$

$$x_i = a + i\Delta x$$

$$= 0 + i\frac{\pi}{n}$$

$$\sum_{i=1}^{n} \sin\left(\pi \frac{i}{n}\right) \cdot \underbrace{\frac{\pi}{n}}_{} = \sum_{i=1}^{10} \sin\left(\frac{i\pi}{10}\right)\frac{\pi}{10}$$



$$\Delta x = \frac{b-a}{n}$$

$i=0 \quad =0$

$i=n$

$$x_i = a + \underbrace{i \cdot \Delta x}$$

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\sum_{i=1}^{n} f(a+i\Delta x)\cdot \Delta x$$

$$\sum_{i=1}^{n} f\left(a+i\,\frac{b-a}{n}\right)\cdot \frac{b-a}{n}$$

Right
$i=1$

$$\sqrt{9 - x^2} = \frac{3\pi}{4}$$

$$9 - x^2 = \left(\frac{3\pi}{4}\right)^2$$

$$9 - \left(\frac{3\pi}{4}\right)^2 = x^2$$

$$\pm\sqrt{9 - \left(\frac{3\pi}{4}\right)^2} = x$$