MAT 126.01, Prof. Bishop, Thursday, Sept. 22, 2020
Review for Midterm 1
Sections 1.1 to 1.5 of textbook
Quizzes 2 and 3

Write the correct answer in the box.

1. Evaluate $\sum_{k=1}^{3} 3^k$

$$= 3^1 + 3^2 + 3^3$$

$$= 3 + 9 + 27$$

$$= 39$$

2. Write in Sigma notation: $\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2n}$.

$$\boxed{\sum_{k=1}^{n} \frac{1}{2k}}$$

$k=1 \quad \frac{1}{2}$

$k=2 \quad \frac{1}{4}$

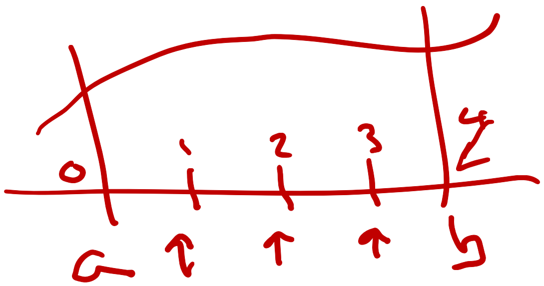
$\vdots$

$k=n \quad \frac{1}{2n}$

$$\frac{1}{4} + \frac{1}{6} + \cdots \frac{1}{2n}$$

$$\sum_{k=2}^{n} \frac{1}{2k}$$

3. Give the Riemann sum approximation to $\int_0^{\pi} \cos(x)dx$ using 4 subintervals and right hand endpoints.



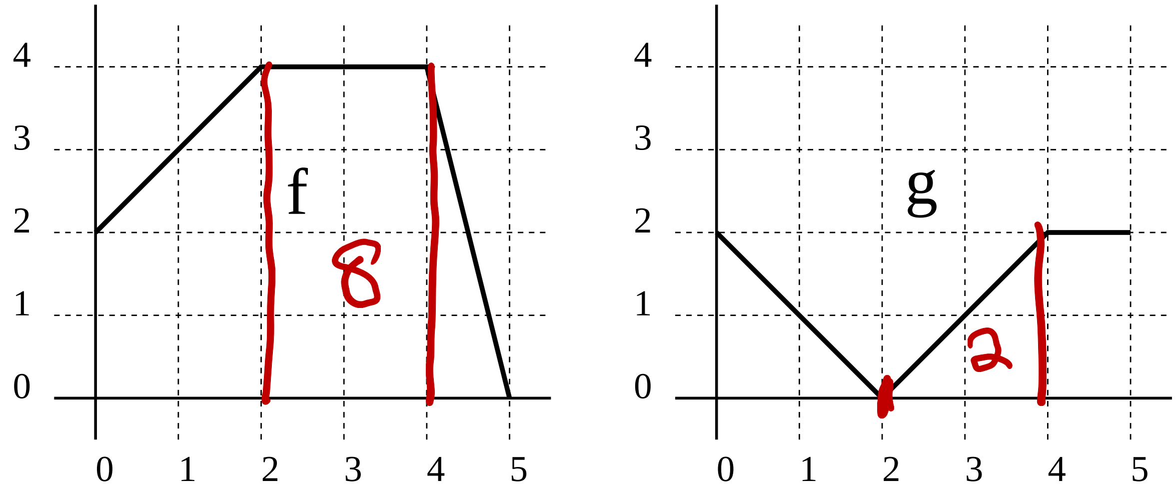
$$\sum_{k=1}^{4} f(a+k\Delta x)\cdot \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$a=0 \quad b=\pi \quad n=4 \quad \Delta x = \frac{\pi-0}{4} = \pi/4$

$$\boxed{\sum_{k=1}^{4} \cos\left(0+k\cdot\frac{\pi}{4}\right)\cdot\underbrace{\frac{\pi}{4}}_{\Delta x}}$$

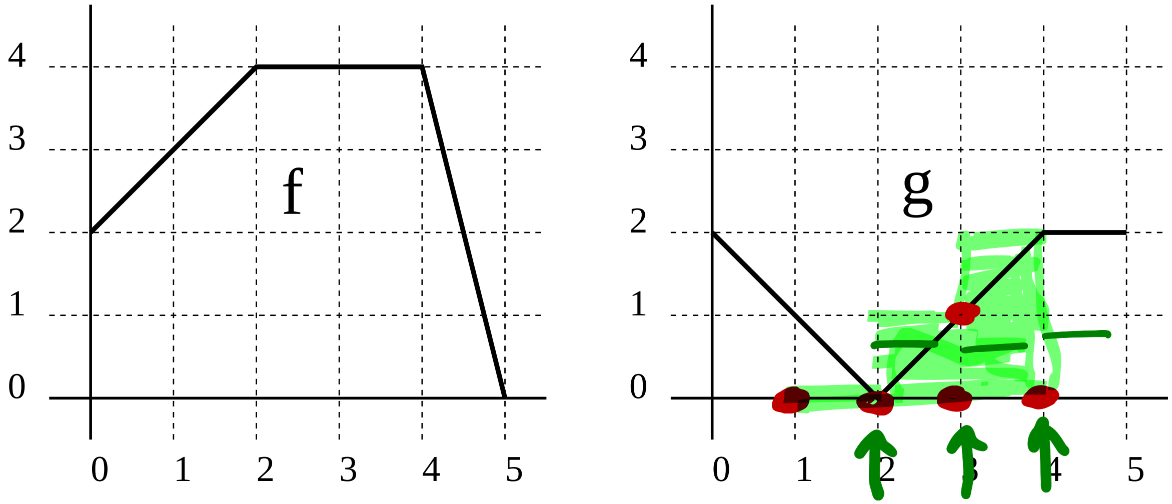
left $\sum_{k=0}^{3}$!



4. Compute the integral $\int_0^5 f(x)dx$ for f plotted above. 16

5. Compute the integral $\int_2^4 f(x) - g(x)dx$ using the functions plotted above.

$= 8 - 2 = 6$

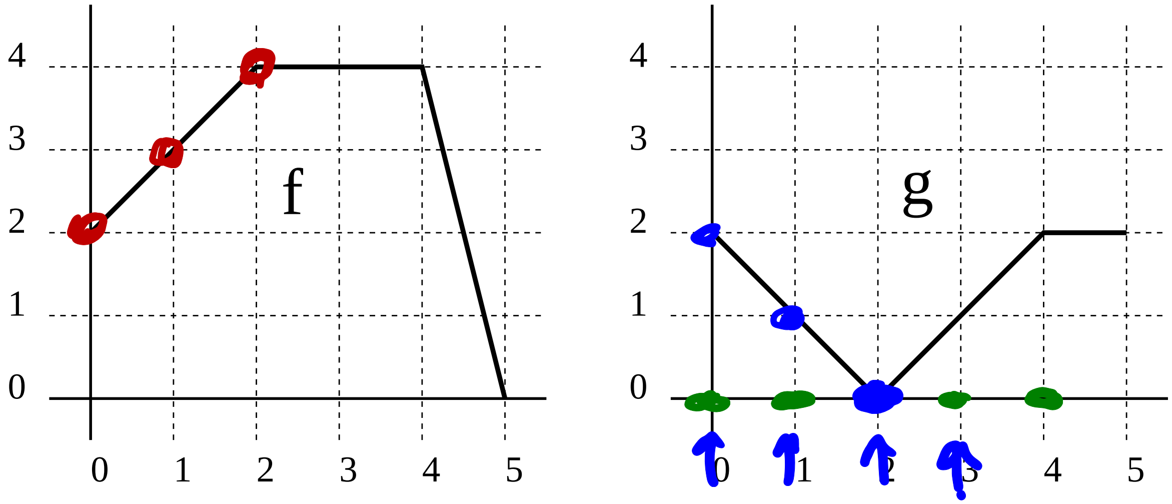


6. Estimate the integral $\int_1^4 g(x)dx$ using the function plotted above and right-hand rule with 3 intervals.

$$\approx \sum \Delta x \cdot g(x_i)$$

$$= 1\cdot 0 + 1\cdot 1 + 1\cdot 2$$

$$= 0 + 1 + 2 = 3$$

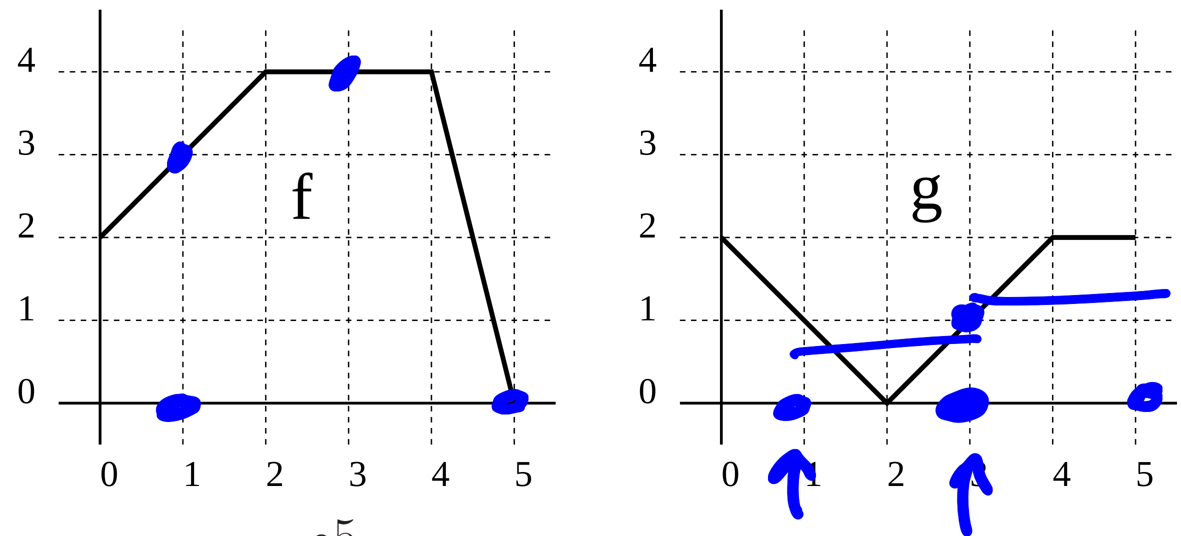


7. Estimate the integral $\int_0^4 g^2(x)dx$ using the function plotted above and left-hand rule with 4 interval (use left endpoint of each subinterval).

$\approx g(0)^2 + g(1)^2 + g(2)^2 + g(3)^2$

$= 2^2 + 1^2 + 0^2 + 1^2$

$= 6$



8. Estimate the integral $\int_1^5 g(f(x))dx$ using the functions plotted above and left-hand rule with 2 intervals.

$\Delta x = 2$

$\approx 2 \cdot g(f(1)) + 2 \cdot g(f(3))$

$= 2 \cdot g(3) + 2 \cdot g(4)$

$= 2 \cdot 1 + 2 \cdot 2 = 6$

~~9. Write down the integral that is represented by~~

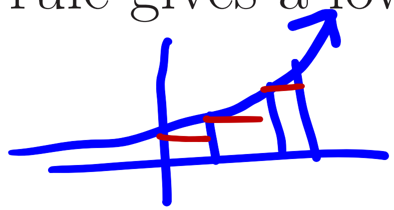
$$\cancel{\lim_{n\to\infty} \frac{2}{n} \sum_{k=1}^{n} \left(4 - \frac{4k^2}{n^2}\right)^{1/2}.}$$

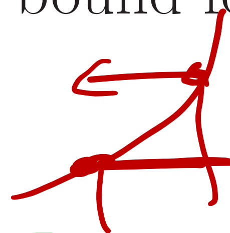
Not on midterm!

~~10. Use geometry to evaluate the integral.~~

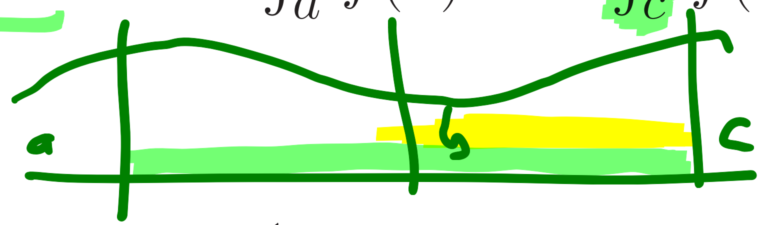
TRUE/FALSE: put a T or F in each box.

11. $\boxed{T}$ The left-hand rule gives a lower bound for $\int_0^4 e^x dx$.





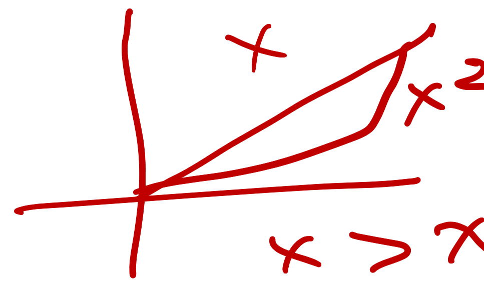
12. $\boxed{T}$ If $a < b < c$ then $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$.



$= -\int_b^c f(x)dx$

13. $\boxed{T}$ $\int_0^1 \sqrt{1+x}dx \geq \int_0^1 \sqrt{1+x^2}dx$.

$0 < x < 1$

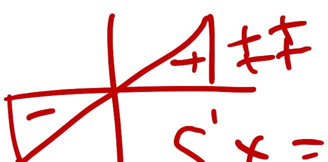


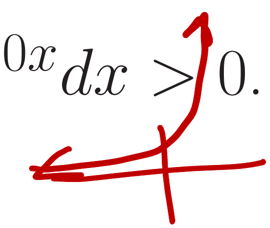
$x > x^2$

$1+x > 1+x^2$

$\sqrt{1+x} > \sqrt{1+x^2}$

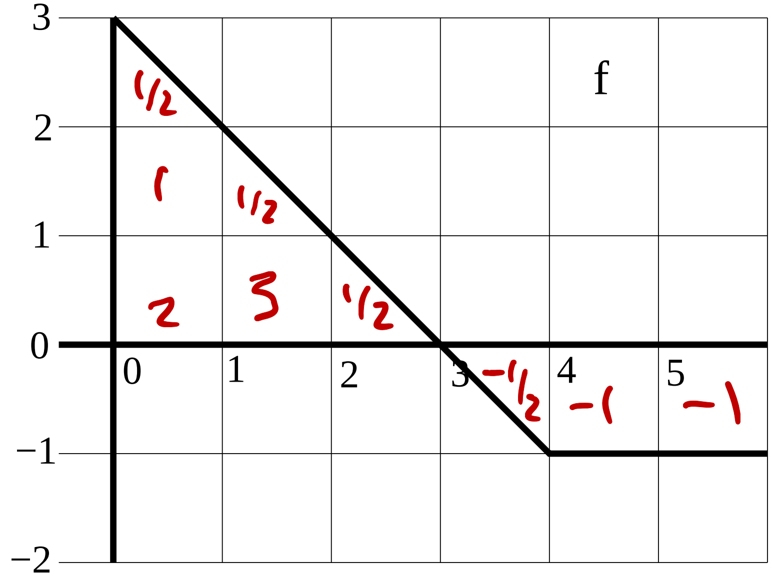
14. $\boxed{T}$ $\int_{-1}^1 xe^{1000x}dx > 0$.





$\int_{-1}^1 x = 0$

$F(x) = \int_0^x f(t)dt$ where f is given by the following figure:



$4\frac{1}{2} - 2\frac{1}{2}$
$= 2$

1. What is $F'(3)$? By FTC $F'(3) = f(3) = 0$

2. What is $F(6) - F(0)$? $= \int_0^6 f(x)dx = 2$

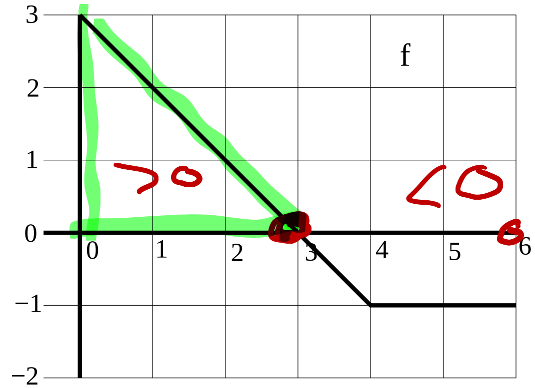
$F(0) = \int_0^0 f\, dx = 0$

f

F max

min

min value.



$F(0), F(6)$

$F(0)=0, \; F(6) = 2$

3

3. At what point x in $[0, 6]$ does F take its maximum value?

$F' = f$

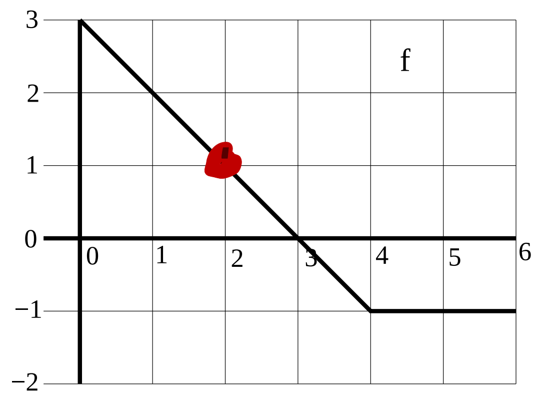
increasing $[0, 3]$

dec $[3, 6]$

3

4. What is the maximum value of F on $[0, 6]$?

$$F(3) = \int_0^3 f\,dx = 9/2 = 4.5$$



$$G(x) = \int_0^{\sin x} f$$

$$G(x) = F(\sin x)$$

5. If $G(x) = \int_0^{2x} f(t)dt$, what is $G'(1)$?

$x = 1$

$$F(x) = \int_0^x f(t)\,dt$$

Chain Rule

$$\boxed{G(x) = F(2x)}$$

$$G'(x) = F'(2x)\cdot(2x)' = F'(2x)\cdot 2$$

$$= f(2x)\cdot 2$$

$$= f(2)\cdot 2 \leftarrow x = 1$$

$$= 1\cdot 2$$

$$= 2$$

$F' = f$

6. A baseball thrown upwards at 96 ft/sec has a velocity given by $v(t) = 96 - 32t$. If it starts at height zero, what is it's height as a function of t?

6

$$v(t) = 96 - 32t$$

$$h(t) = \underline{h(0)} + \int_0^t 96 - 32s\,ds$$

Net change thm

$$= 0 + \left(96 - 32 \cdot \tfrac{1}{2} s^2\right)\Big|_0^t$$

$$= \underline{96t - 16t^2}$$

7. If f is given by the figure on the right, which of the following is the largest?

$\int_0^1 f(x)dx$

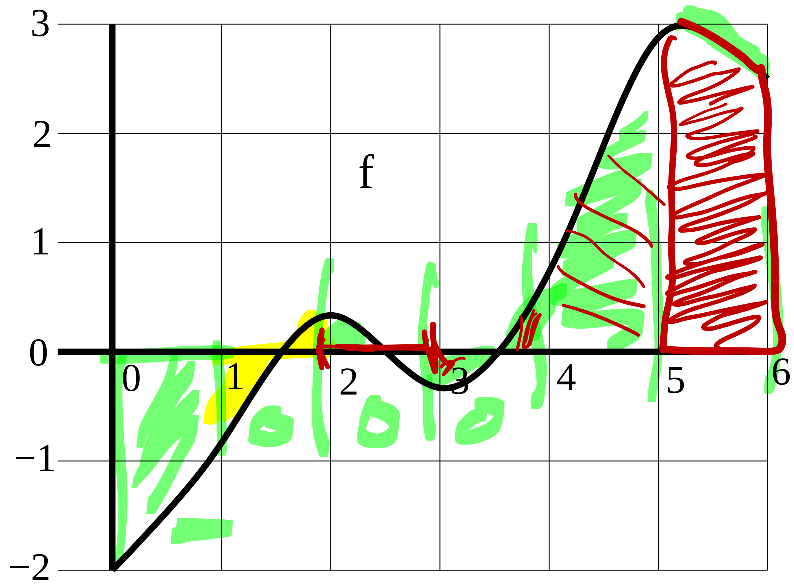
$\int_1^2 f(x)dx$

$\int_2^3 f(x)dx$

$\int_3^4 f(x)dx$

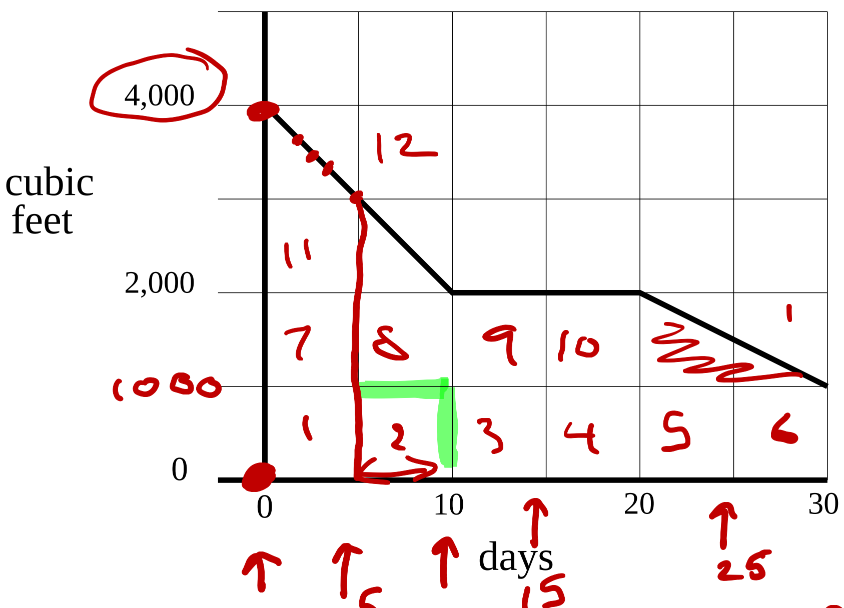
$\int_4^5 f(x)dx$

$\int_5^6 f(x)dx$



\$3, \$1, \$5

8. A warehouse charges its customers \$2 per day for every cubic foot of space used for storage. The figure on the right shows the storage used by one company over a month. How much will the company have to pay?



13 boxes under f

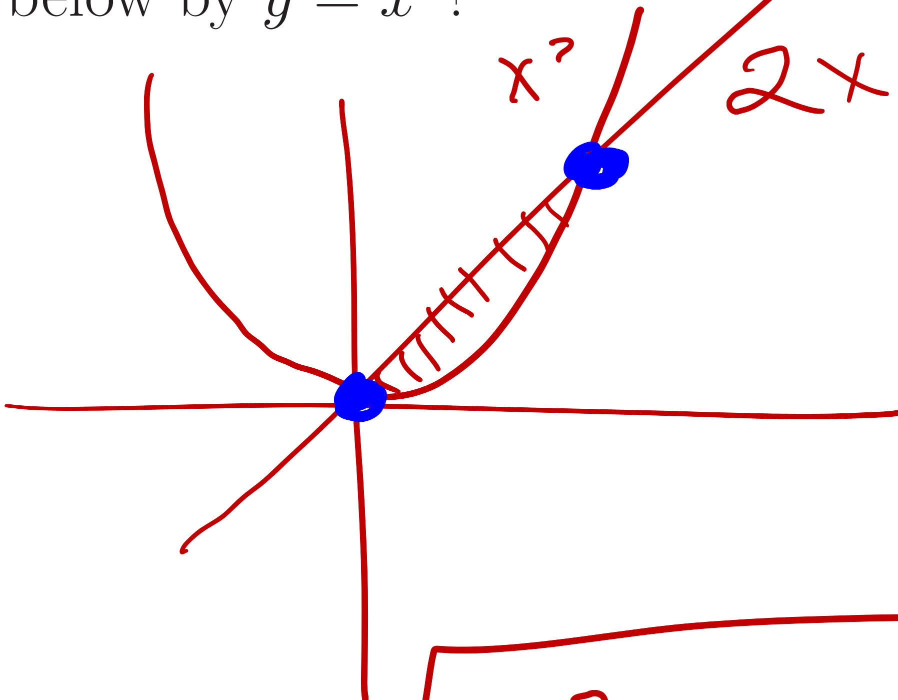
13 × \$10,000

= \$130,000

box = 5000 ft^3

= \$10,000 cost.

9. Which integral gives the area of the region bounded above by $y = 2x$ and below by $y = x^2$?



$2x = x^2$?

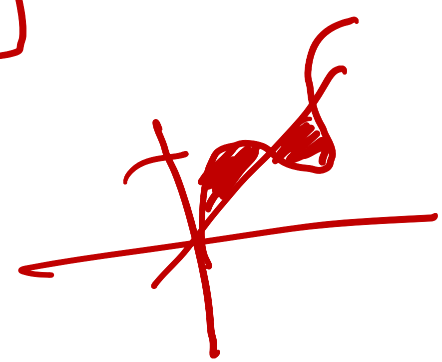
$0 = x^2 - 2x$

$= x(x-2)$

$x = 0, \quad x = 2$

$$\boxed{\int_0^2 2x - x^2 \, dx}$$

$f > g \qquad \int f - g$



10. $\boxed{d}$ Taking $u = x^2 + 1$ allows you to easily evaluate which of the following integrals?

$du = 2x\,dx$

(a) $\int x^2 \cos(x^2 + 1)dx$

(b) $\int \sin(x^2 + 1)dx$

(c) $\int \frac{x^2-1}{x^2+1}dx$

(d) $\int xe^{x^2+1}dx$

(e) $\int (x - 1)\sqrt{x^2 - 1}dx$

(f) $\int \sqrt{x^2 + 1}dx$

(g) $\int \ln(x^2 - 1)dx$

not on Midterm

11. ~~Evaluate~~ $\sum_{k=0}^{3} 3^k$

12. Evaluate $\sum_{k=3}^{10} k = 3 + 4 + 5 + \cdots + 10 = 55 - 1 - 2$

$= 52$

$1 + 2 + \cdot$

$$\boxed{\sum_{1}^{n} k = \frac{(n+1)(n)}{2}}$$

Given on exam

$$\sum_{1}^{10} = \frac{11 \cdot 10}{2} = 11 \cdot 5 = 55$$

$\sum_{1}$ $\sum_{2}$ $\sum_{3}$ $\boxed{\sum_{1}^{12} k}$ $\boxed{\sum_{4}^{12} k}$

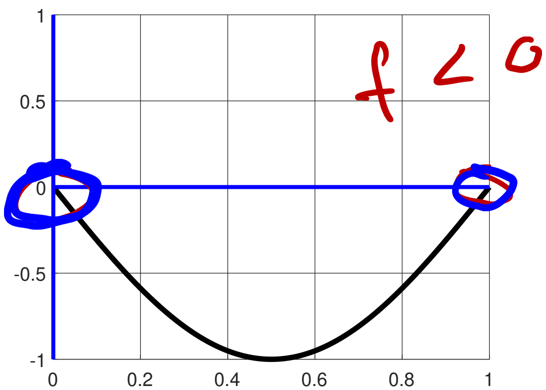
$-1-2-3$

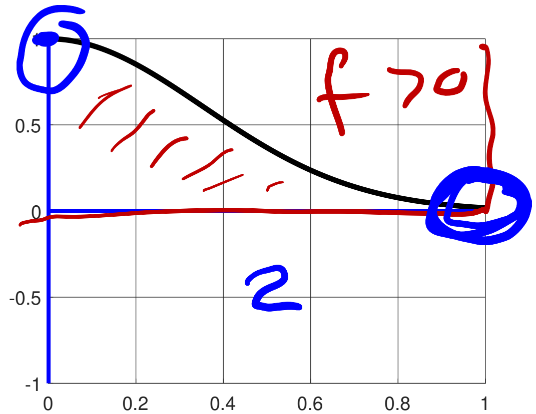
13. Give the Sigma notation for Match the sum to the correct formula: $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots+\frac{1}{n^2}$,

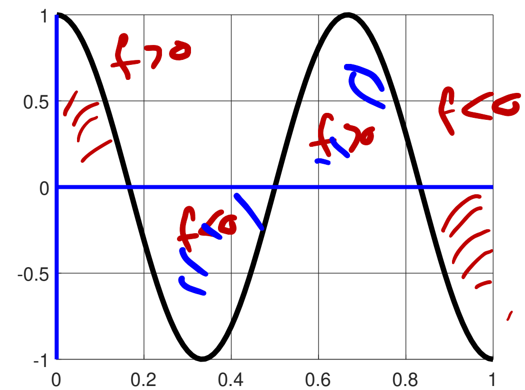
14. Given the formula for left hand approximation of $\int_2^4 x^3 dx$ with four subintervals.

$F' = f$

15. For each function f graphed on $[0, 1]$, sketch a graph of $F(x) = \int_0^x f(t)dt$.

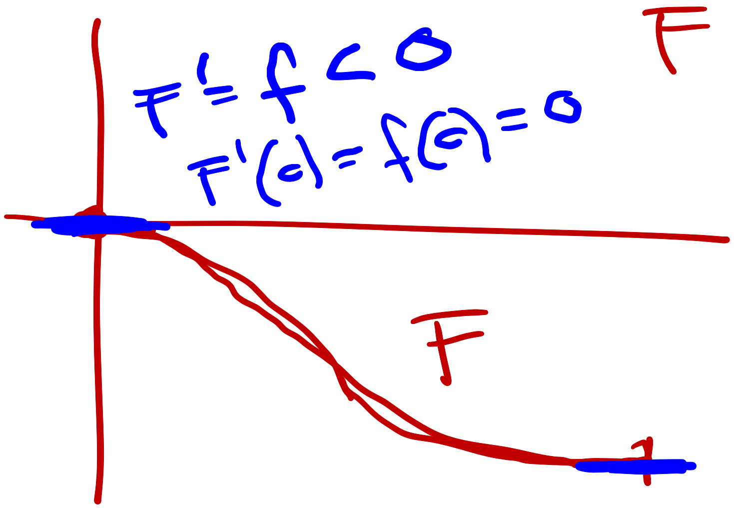


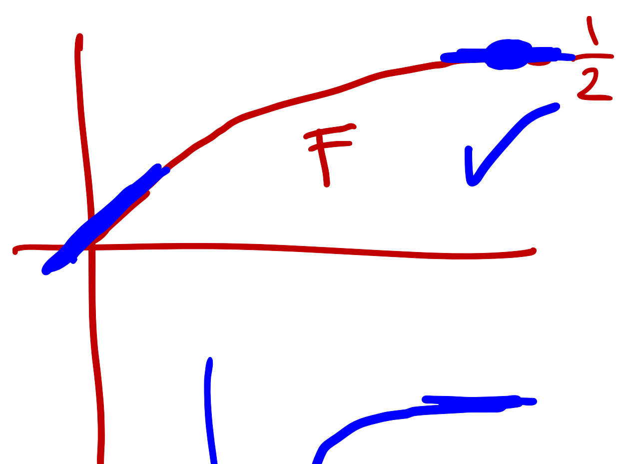


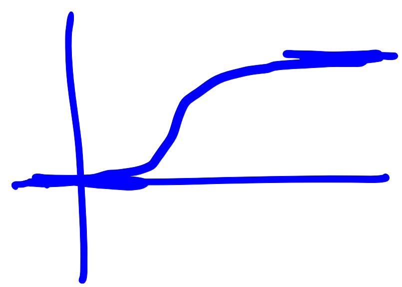


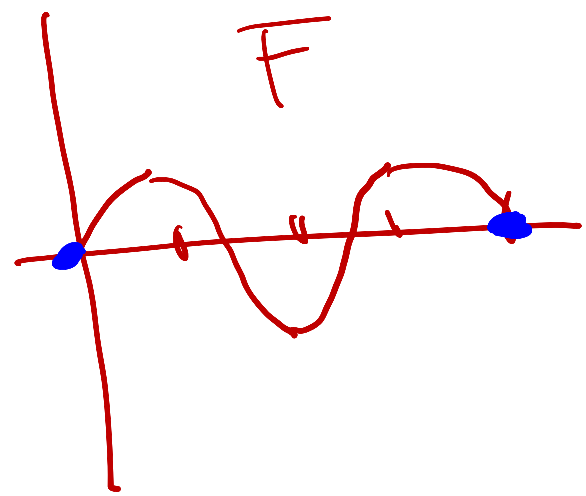
$F' = f < 0$

$F'(0) = f(0) = 0$









16. Find $\int \sin(x)\sqrt{2+\cos(x)}\,dx$

$u = 2 + \cos x$

$du = -\sin x \, dx$

$-\int \underbrace{\sqrt{2+\cos x}}_{u} \underbrace{(-\sin x)\,dx}_{du}$

$= -\int \sqrt{u}\,du$

$= -\frac{2}{3} u^{3/2} + C$

$= -\frac{2}{3}(2+\cos x)^{3/2} + C$

3 def $\int_a^b$

3 indef $\int$

Subst.
power
exp
sin, cos, ...

$\int \frac{1}{x} = \ln$

$\int \cos^3$

$\boxed{\cos^2 = 1 - \sin^2}$

Know this

$\frac{1}{x}$, $1+e^x$

17. Find $\int \frac{e^x}{1+e^x}dx$

What to choose for u?

$\int f(\underbrace{g(x)}_{u=g(x)})\, g'(x)$

$\frac{1}{1+e^x} \cdot e^x dx$

$u = 1+e^x \quad du = e^x dx$

$= \int \frac{1}{u} du = \ln u + C$

$= \ln(1+e^x) + C$ ✓

3 odd powers of sin or cos.

18. Find $\int \cos^5(x)dx$

$\int \cos^3 = \int \cos(1-\sin^2) = \int \cos$ ✓ $- \int \sin^2 \cos$ ✓

$\boxed{\cos^2 = 1 - \sin^2}$

$\int \underline{\cos^1} (\cos^2)^2$

$= \int \cos (1-\sin^2)^2$

$= \int \cos (1 - 2\sin^2 + \sin^4)$

$= \int \cos - 2 \int \underbrace{\sin^2 \cdot \cos}_{u = \sin \quad u^2} + \int \underbrace{\sin^4 \cdot \cos}_{u=\sin \quad u^4} dx$

$= \underbrace{\int \cos}_{}$ ✓

19. Find $\int_0^1 \underbrace{x^3}_{}(\underbrace{x^4 + 1}_{})^4 dx$

$u = x^4 + 1$

$du = 4x^3$

$= \frac{1}{4} \int \underline{4x^3} (\underbrace{4^x + 1}_{u})^4 \underline{dx}$

$= \frac{1}{4} \int u^4 \, du$

$= \frac{1}{4} \cdot \frac{1}{5} u^5$

$= \frac{1}{20} (x^4 + 1)^5 \Big|_0^1$

$= \frac{1}{20} (32 - 1) = \frac{31}{20}$

20. Find $\int_0^1 x^n(1+x^n)dx$

$n \neq -1 \quad n \neq 1/2$

$$= \int_0^1 x^n + x^{2n}$$

$$= \frac{1}{n+1} x^{n+1} + \frac{1}{2n+1} x^{2n+1} \Big|_0^1$$

$$= \left(\frac{1}{n+1} + \frac{1}{2n+1} \right) - (0 - 0)$$

$x = 1$

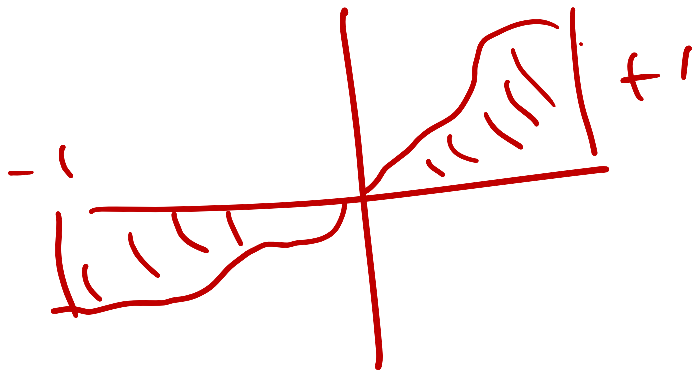
$$= \frac{1}{n+1} + \frac{1}{2n+1}$$

21. Find $\int_{-1}^{1} \sin(x^3)dx$ (this is a trick problem). $= 0$

$\sin(x^3) =$ odd

$\sin((-x)^3) = \sin(-x^3)$

$= -\sin(x^3)$



Office Hours Sept 22

start $\approx$ 11:30