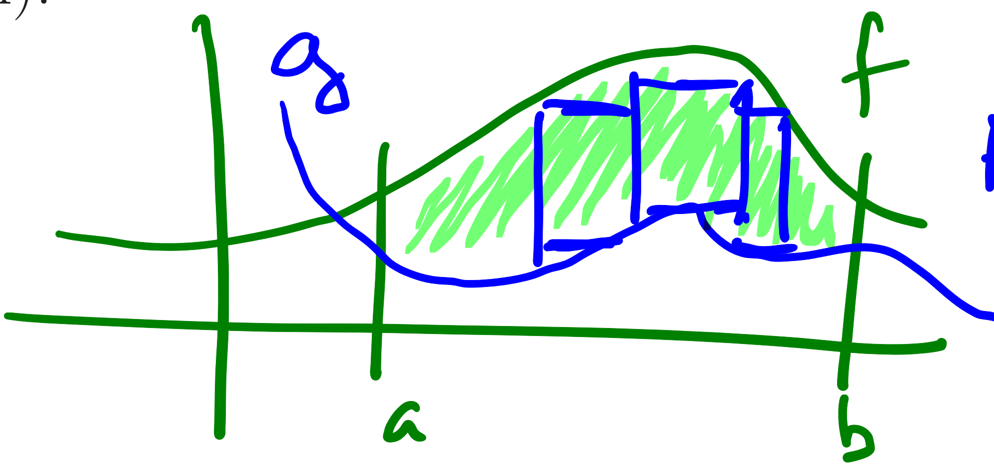
MAT 126.01, Prof. Bishop, Thursday, Sept. 24, 2020
Section 2.1, Area between curves

If $g \leq f$ on $[a, b]$ then the area between the graphs of f and g and the lines $x = a$, $x = b$ is

$$\int_a^b f(x) - g(x) dx.$$

If $f < g$ we have to reverse summation.

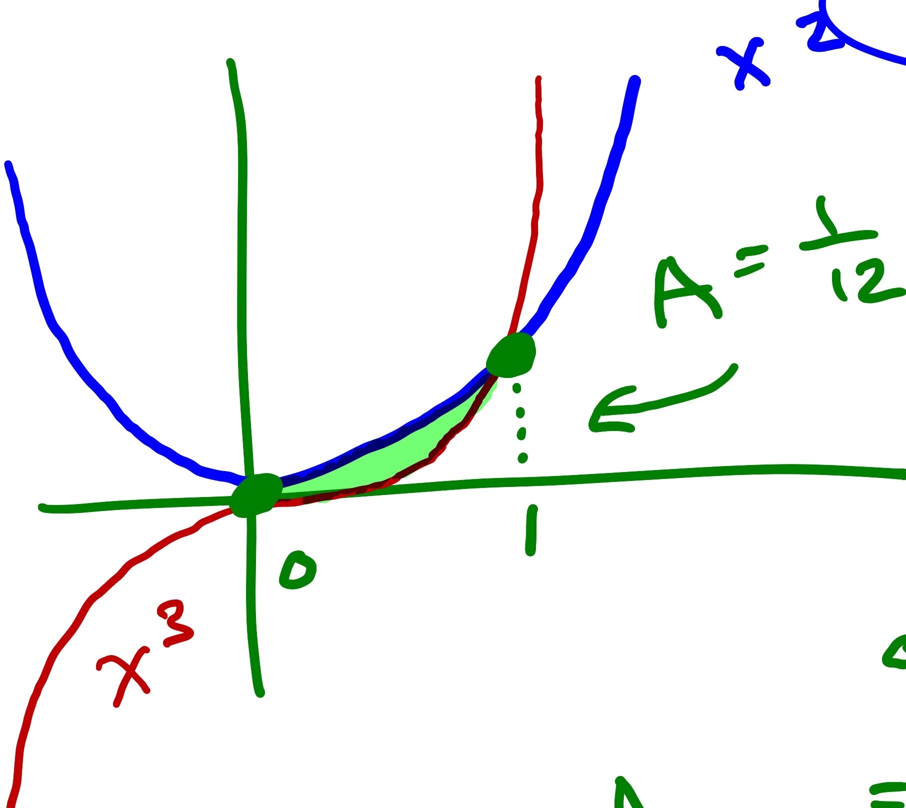
If f and g cross inside $[a, b]$ we have to break integral into pieces (compound region).



$$A = \int_a^b f(x) - g(x) dx$$

$$\int g - f < 0$$

Find area between x^2 and x^3 for $0 \le x \le 1$.



$x^2 = x^3$

$0 = x^3 - x^2$

$= x^2(x-1)$ $\qquad x^2 = 0$, $\quad (x-1) = 0$

$x=0$, $\quad x=1$

on $[0,1]$ $\quad x^2 > x^3$

$0 < x < 1$

$x^3 < x \cdot x < x$

$x > 1 \quad x^2 > x$

$$\text{Area} = \int_0^1 x^2 - x^3 \, dx$$

$$= \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1 = \frac{1}{3} - \frac{1}{4} - 0$$

$$= \frac{1}{12}$$

$0 < x < 1$

$x > x^2 > x^3 > \cdots$

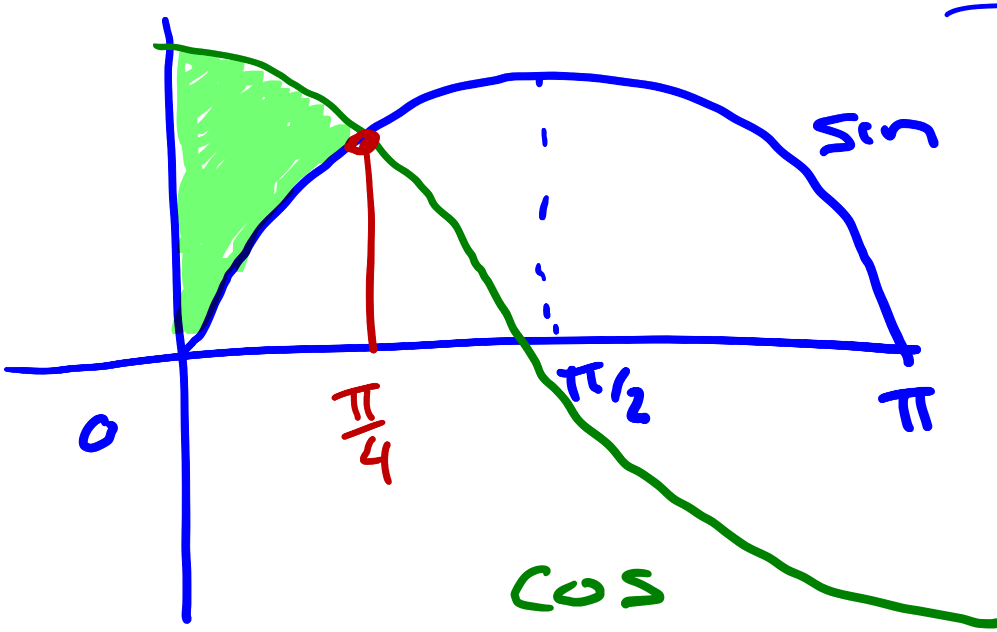
$1/2 > 1/4 > 1/8 > \cdots$

$x > 1$

$x < x^2 < x^3 < \cdots$

$2 \quad 4 \quad 8$

Find area between $\sin x$ and $\cos x$ for $0 \le x \le \pi/4$.



$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \cos(\pi/4)$$

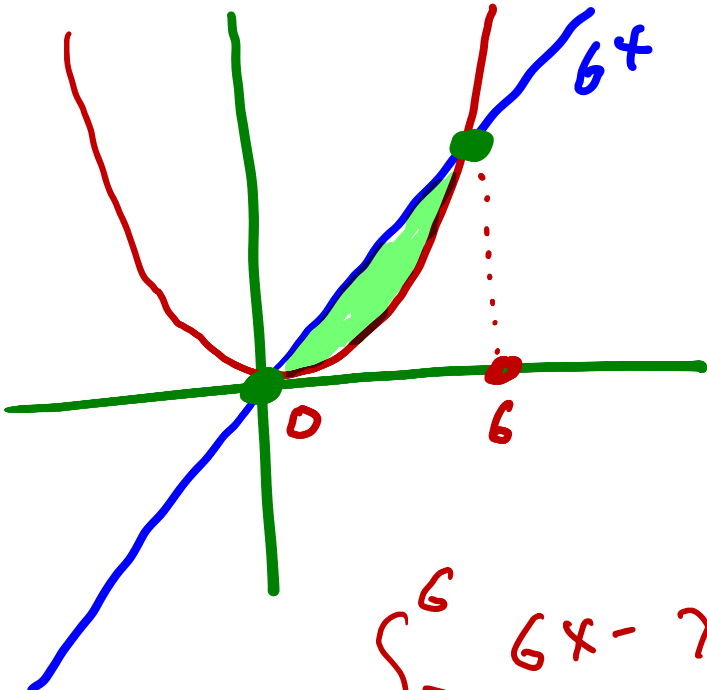
$$A = \int_0^{\pi/4} \cos(x) - \sin(x)\, dx = \sin(x) + \cos(x) \Big|_0^{\pi/4}$$

$$= \left(\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)\right) - \left(\sin(0) + \cos(0)\right)$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0 + 1)$$

$$= \frac{2}{\sqrt{2}} - 1 = \frac{2 - \sqrt{2}}{2} > 0$$

Find area of region ~~between~~ below $6x$ and above x^2.
First find where these graphs cross.



$$6x = x^2$$

$$0 = x^2 - 6x$$

$$= x(x-6)$$

$$x = 0, \quad x = 6$$

$$\int_0^6 6x - x^2\,dx = 6\cdot\frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^6$$

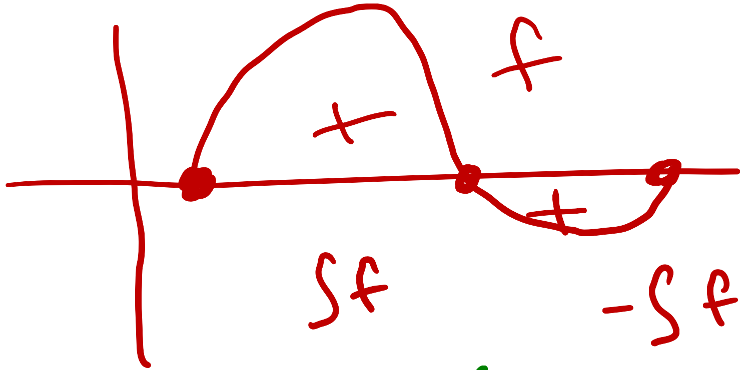
$$= 3x^2 - \frac{1}{3}x^3\Big|_0^6$$

$$= \left(3\cdot 36 - \frac{1}{3}6^3\right) - (0-0)$$

$$= (3\cdot 36) - \frac{1}{3}\cdot 6\cdot 36 = 3\cdot 36 - 2\cdot 36$$

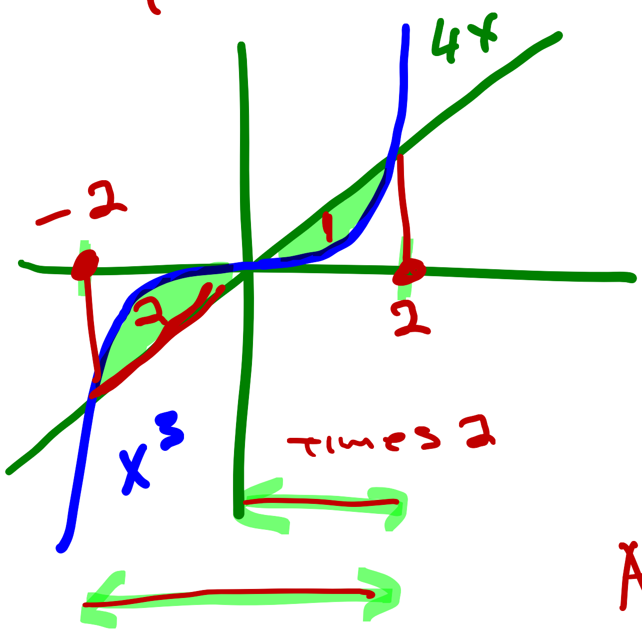
$$\boxed{= 36}$$

Find the total area of region between $4x$ and above x^3.
Total area = all regions count as positive.



Total area = all +

area = + and −



$$4x = x^3$$
$$0 = x^3 - 4x$$
$$= x(x^2-4)$$
$$= x(x-2)(x+2)$$
$$x=0,\ x=2,\ x=-2$$

$$A = 2\int_0^2 4x - x^3 = 2\left[2x^2 - \frac{1}{4}x^4\right]_0^2$$
$$= 2[8-4] - [0-0]$$
$$= \boxed{8}$$

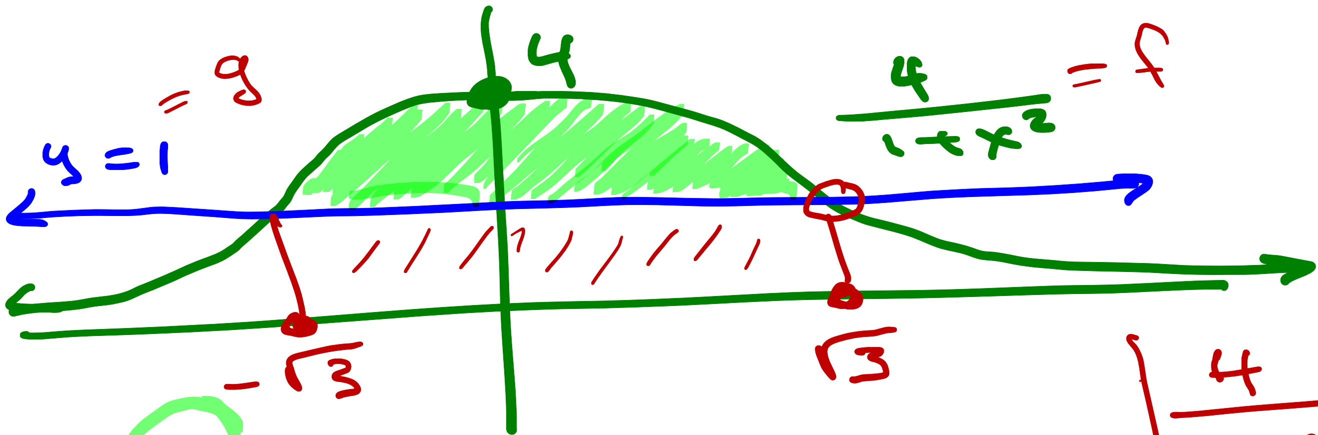
$$A = \int_{-2}^{0}(x^3-4x)\,dx + \int_0^2 4x - x^3\,dx$$

Find area of region $\{(x, y) : 1 \le y \le 4/(1 + x^2)\}$.

g f

$$\frac{4}{1+x^2}$$

$$x=0$$



$$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4}{1+x^2} - 1 \, dx$$

$$= 4\int_{-\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2} - \int_{-\sqrt{3}}^{\sqrt{3}} 1 \, dx$$

$$\boxed{= 2\int_0^{\sqrt{3}}}$$

$$= 4 \tan^{-1}(x) \Big|_{-\sqrt{3}}^{\sqrt{3}} - 2\sqrt{3}$$

$$= 4\left(\tan^{-1}(\sqrt{3}) - \tan^{-1}(-\sqrt{3})\right) - 2\sqrt{3}$$

$$= 4\left(\pi/3 - (-\pi/3)\right) - 2\sqrt{3} \Rightarrow \boxed{\frac{8\pi}{3} - 2\sqrt{3}}$$

$$\frac{4}{1+x^2} = 1$$

$$4 = 1 + x^2$$

$$3 = x^2$$

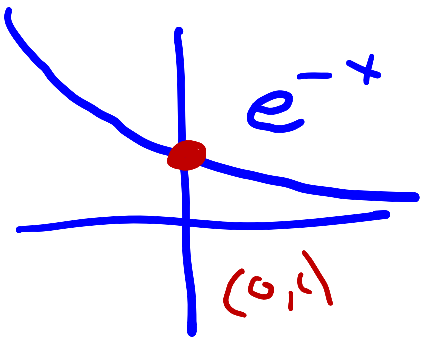
$$\pm\sqrt{3} = x$$

Quiz 4 review:

4 "find the graph"

Find e^{-x} in the graph below. $\boxed{D}$

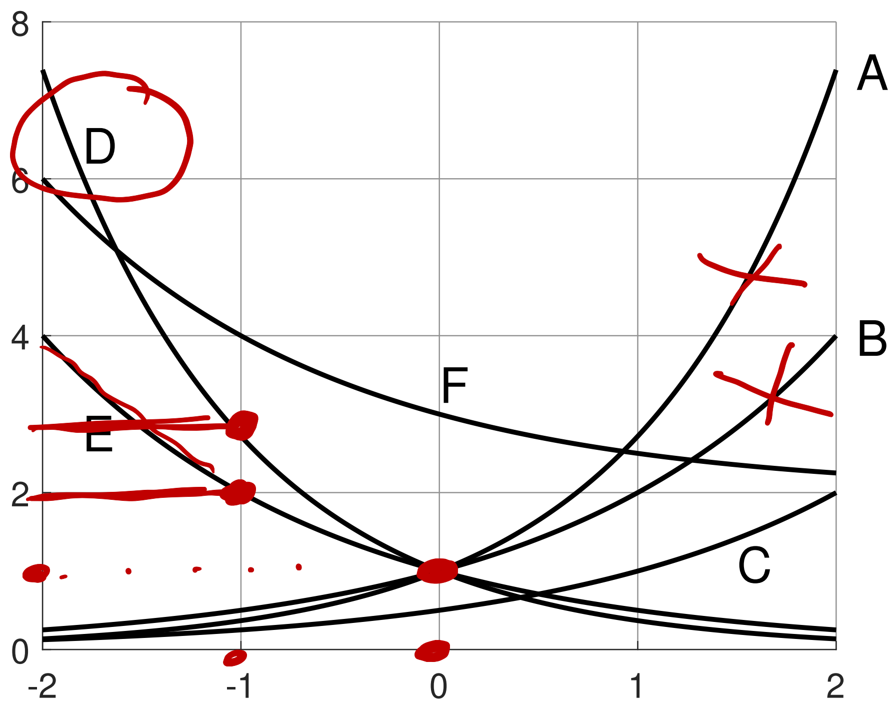
$e > 1$



$e^{-0} = 1$

$e^{-(-1)} = e^{1} \approx 2.7$

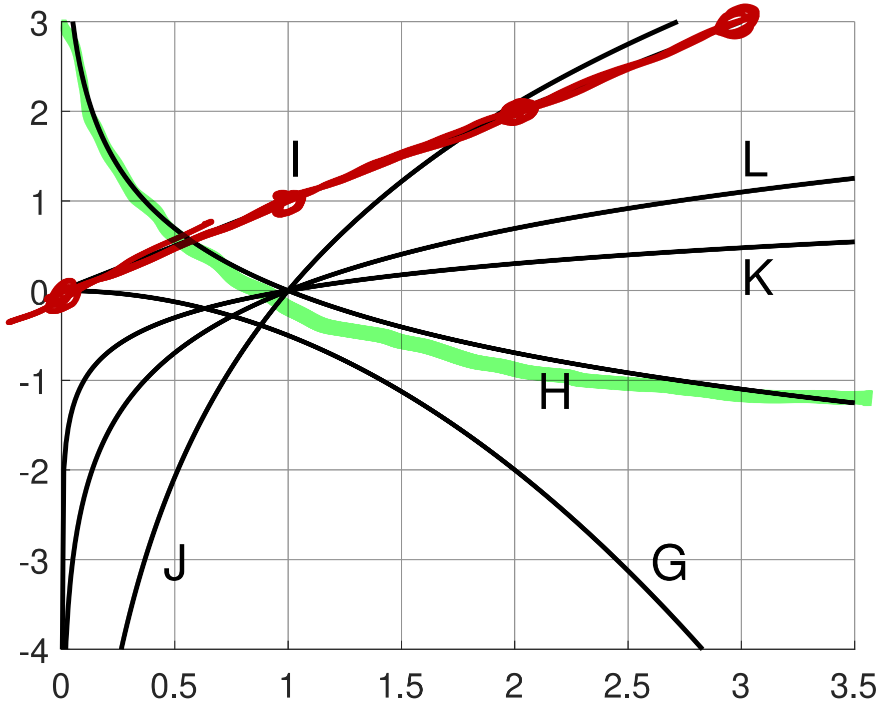
$x = -1$



E $\quad 2^{-x}$

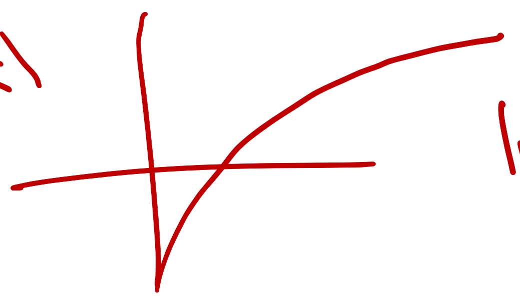
Find $\ln(e^x)$ in the graph below. $\boxed{I}$ $\ln(e^x) = x$

x



$\ln(\frac{1}{x}) = -\ln x$

$\ln(x)$



$\ln(e) = 1$

Compute the area of the shaded region.

$$\int_0^1 \frac{1}{1+x^2} = \tan^{-1}(1) - 0 = \pi/4$$

(a) $\pi/6$

(b) $\pi/4$

(c) $\pi/3$

(d) $\pi/2$

(e) $2\pi/3$

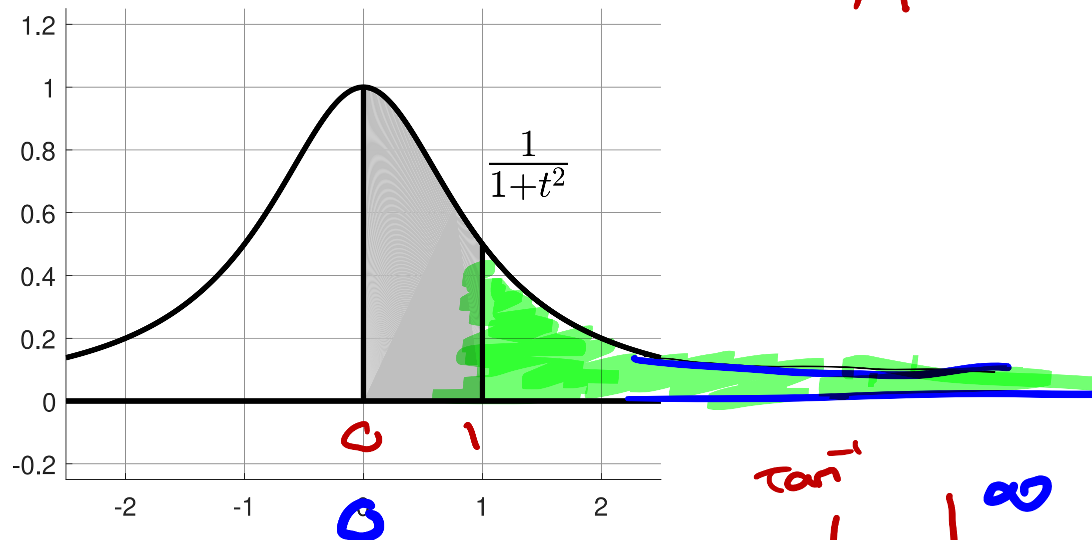
(f) $3\pi/4$

(g) π

(h) $\frac{1}{2}\tan^{-1}(2)$

(i) $\tan^{-1}(2)$

(j) $2\tan^{-1}(2)$

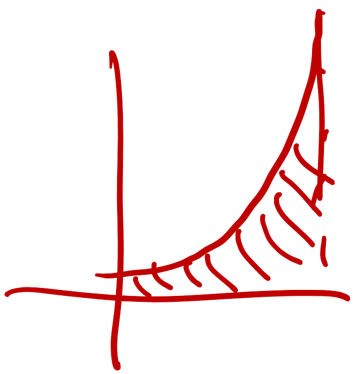


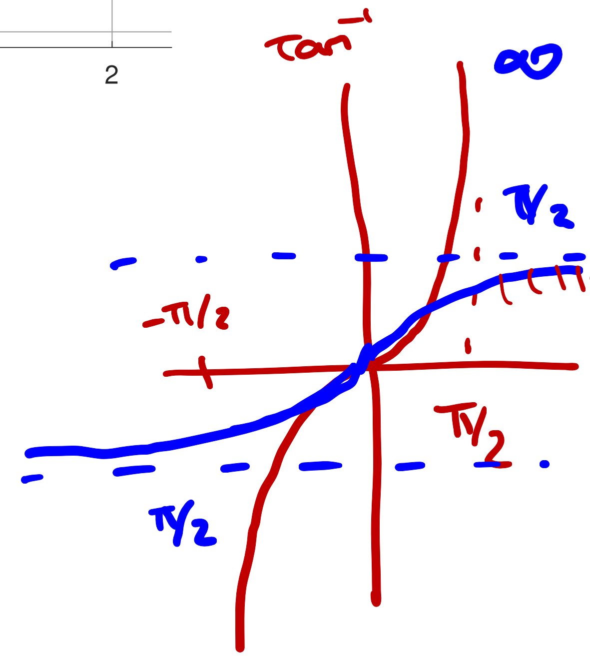
$$\int_0^\infty \frac{1}{1+t^2}\,dt = \tan^{-1}(x)\Big|_0^\infty$$

$$= \tan^{-1}(\infty) - \tan^{-1}(0)$$

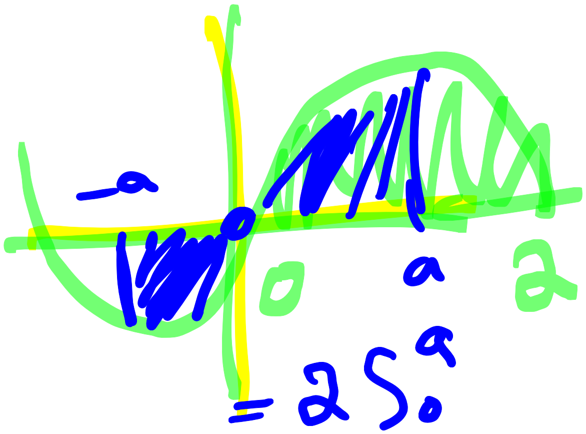
$$= \frac{\pi}{2} - 0$$

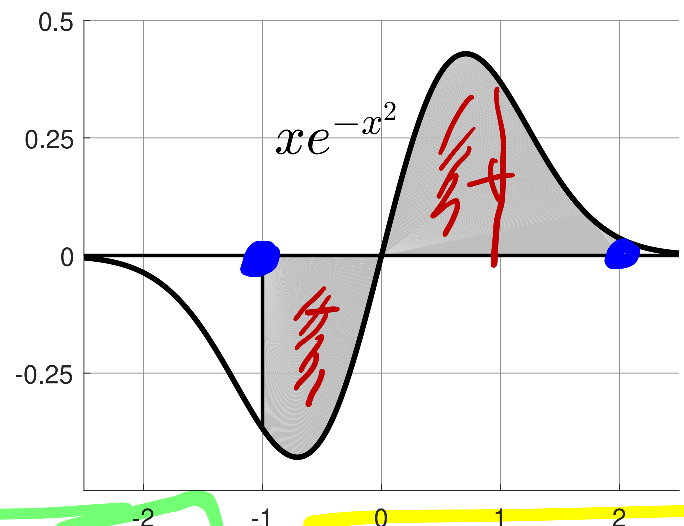
$$= \pi/2$$





Compute the total area of the shaded region (all regions count as positive area).





$\cancel{\int_{-1}^{2}}$

$xe^{-x^2} = \text{odd}$

$$A = \boxed{-\int_{-1}^{0} \left(xe^{-x^2}\right) dx} + \boxed{\int_0^2 x e^{-x^2} dx} = \int_0^2$$

$$u = -x^2 \qquad du = -2x\,dx$$

$$= \frac{-1}{-2} \int_{-1}^{0} (-2x) e^{-x^2} dx$$

$$= \frac{1}{2} \int_{-1}^{0} e^u du = \frac{1}{2} e^u$$

$$= \frac{1}{2} e^{-x^2}\Big|_{-1}^{0} = \boxed{\frac{1}{2}\left[1 - \frac{1}{e}\right]}$$

$$= -\frac{1}{2} \int_0^2 e^u du$$

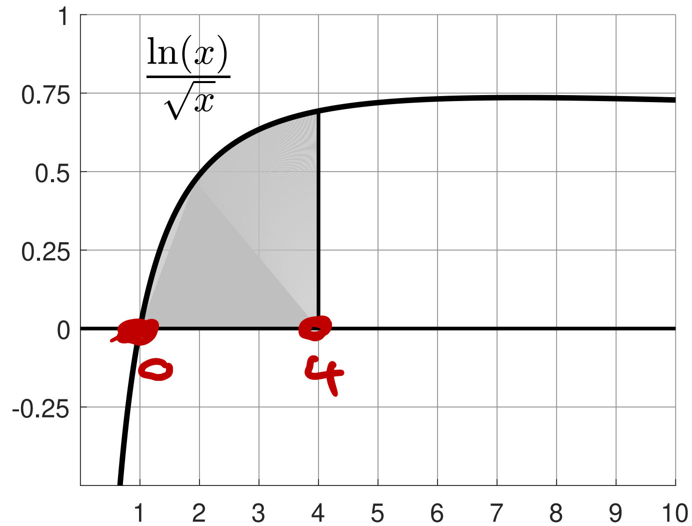
$$= -\frac{1}{2} e^{-x^2}\Big|_0^2$$

$$= -\frac{1}{2}\left[e^{-4} - e^{-0}\right]$$

$$\boxed{= \frac{1}{2}\left[1 - e^{-4}\right]}$$

Compute the area of the shaded region on the right.

(a) $2\ln(2)$
(b) $2\ln(3)$
(c) $4\ln(2)$
(d) $8\ln(2) - 4$
(e) $4\ln(4) - 3$
(f) $12\ln(3) - 8$
(g) $6\ln(3) - 6$
(h) $9\ln(9) - 8$
(i) $8\ln(8) - 4$



$$\int_1^4 \frac{\ln(x)}{\sqrt{x}}\,dx = \int_1^4 x^{-1/2}\underbrace{\ln x}\,dx$$

$$(\ln x) = \frac{1}{x}$$

$$(\ln\sqrt{x})' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2}\frac{1}{\sqrt{x}}$$

Compute the area of the shaded region on the right.

(a) $e^{-a^2} - 1$

(b) e^{-a^2}

(c) $e^{a} - 1$

(d) $\frac{1}{2}(e^{a} - 1)$

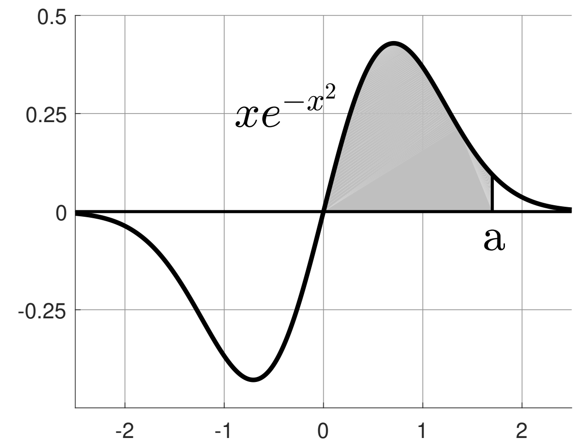
(e) e^{a^2}

(f) $\frac{1}{2}e^{-a^2}$

(g) $\frac{1}{2}(e^{-a^2} - 1)$

(h) $\frac{1}{2}(1 - e^{a^2})$

(i) $\frac{1}{2}e^{-a^2} - 1$



$$\int_0^a x e^{-x^2} dx$$

$$u = -x^2 \quad du = -2x\,dx$$

$$-\frac{1}{2}\int e^u du = -\frac{1}{2}e^{-x^2}\Big|_0^a$$

$$= -\frac{1}{2}e^{-a^2} - \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2}\left[1 - e^{-a^2}\right]$$

Match each formula for the area to the region it describes.

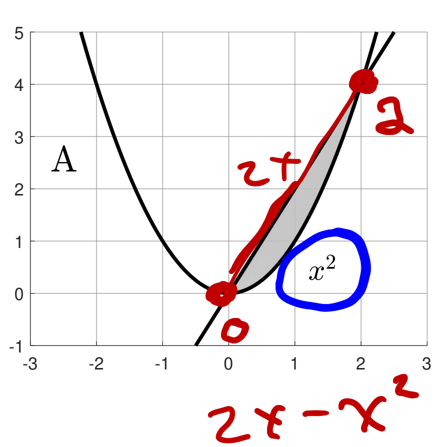
$\int_{-\sqrt{2}}^{\sqrt{2}} \cancel{x^2 dx}$ $2 - x^2 dx$ C

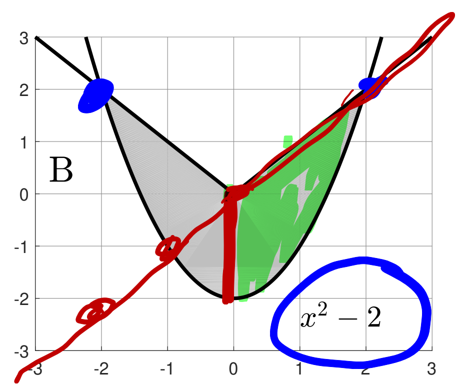
$\boxed{\int_0^2 2x - x^2 dx}$ A

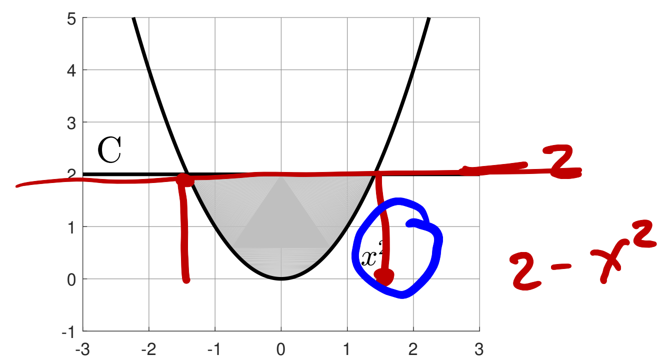
$2\int_0^2 2 + x - x^2 dx$ ✓ C

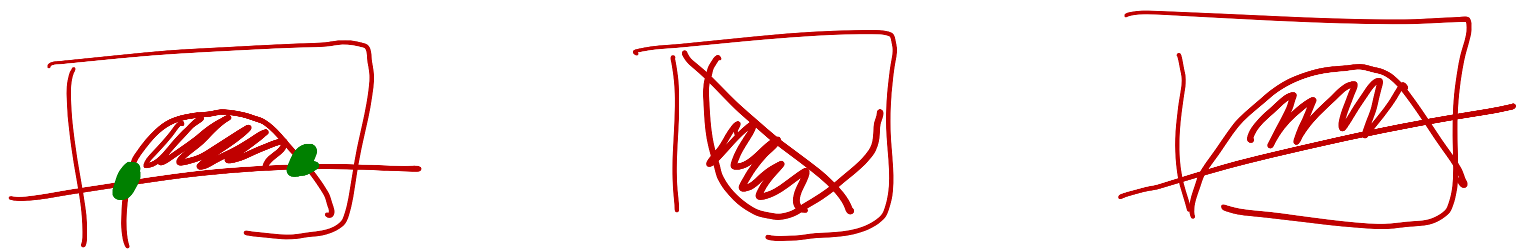
$2 \cdot \int_0^2 x - (x^2 - 2)$

$x + 2 - x^2$









Quiz :

easy? 4 find the graph 1.6

3 do integral 1.7

easy? 3 math formula Area to region 2.1

$\overline{}$

10

$\boxed{\tfrac{1}{3} \text{ over}}$:)

4 week 6

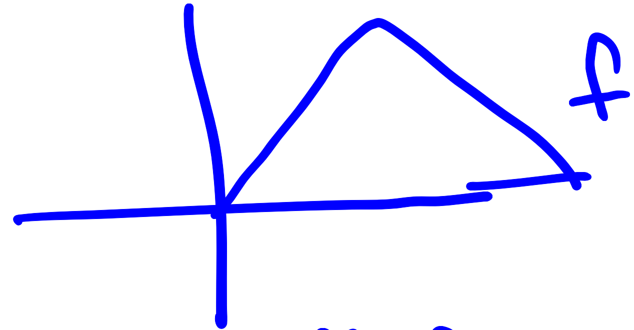
exps, ln, inverse trig.

MAT 126

Office Hours

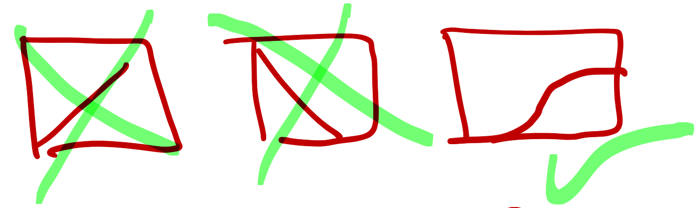
start $\approx$ 11:20

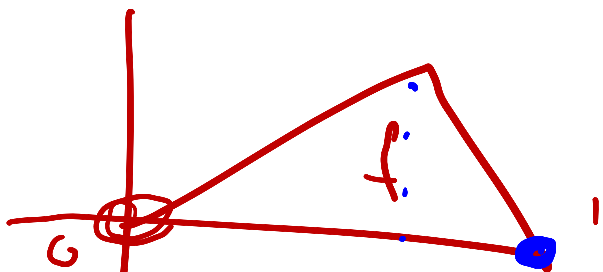
Give you graph f

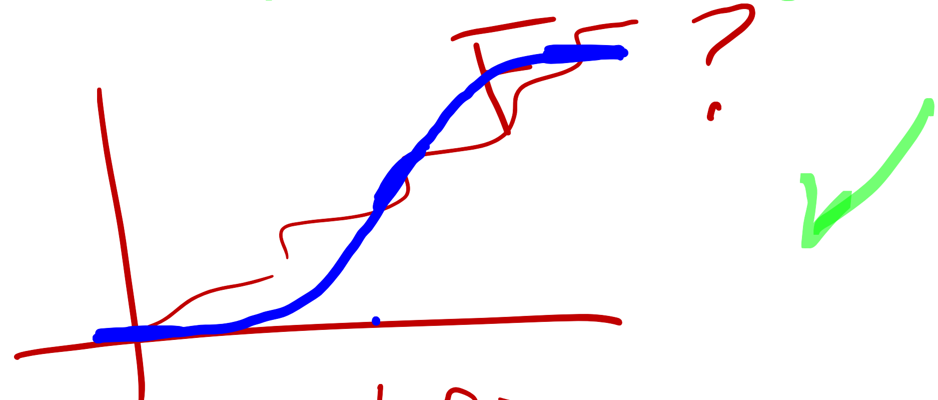


Pick out graph of $F = \int_0^x f$.

By FTC $\underline{F'} = \underline{f}$.





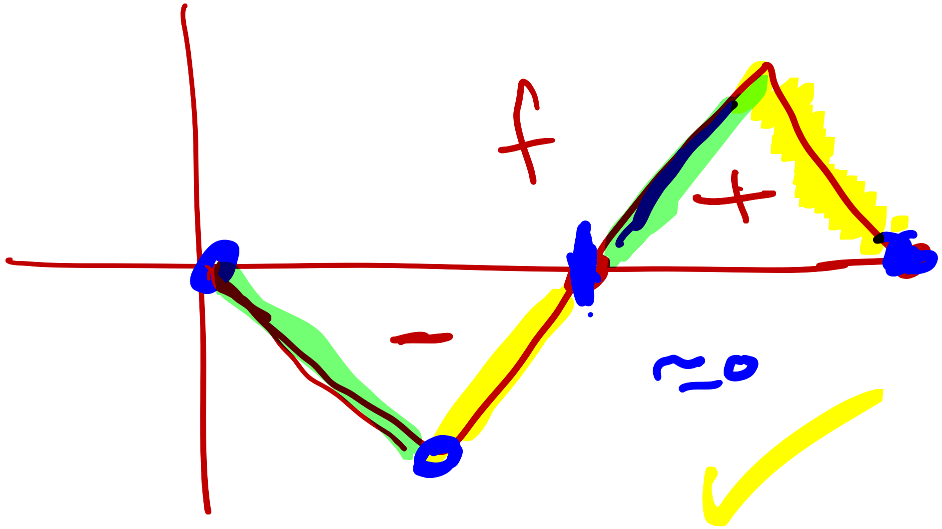


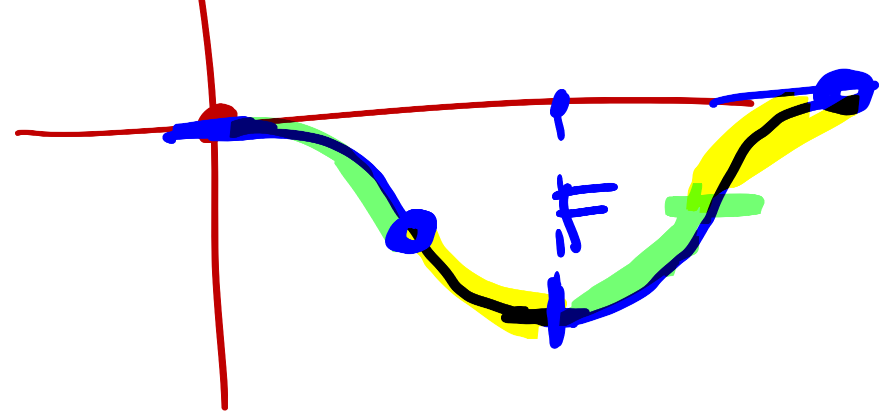
$f > 0 \quad f(0) = 0 \quad f(1) = 0$

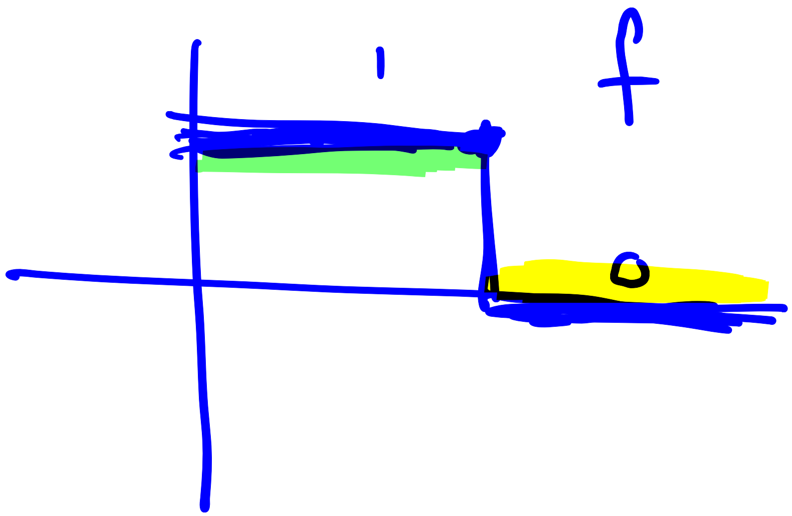
$F' = f > 0$

$F' = f$

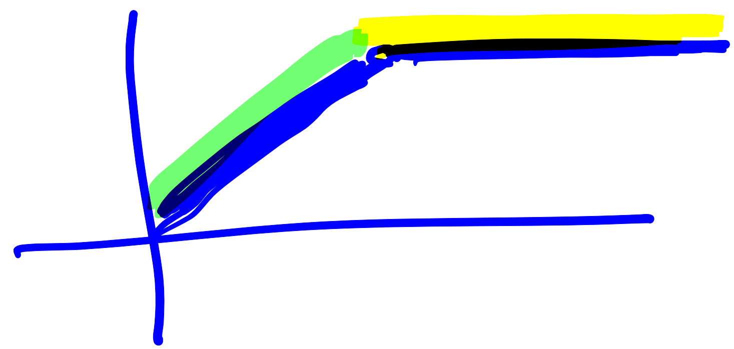
$F = \int f$







$F' = f$



$\int \cos^3$

odd power of cos or sin

$\cos^2 + \sin^2 = 1$

$\boxed{\cos^2 = 1 - \sin^2}$

$$\int \cos^3 = \int \cos \cdot \cos^2$$

$$= \int \cos(1 - \sin^2)$$

$$= \int \cos - \int \sin^2 \cos$$

$u = \sin \quad du = \cos$

$$= \sin - \int u^2 \, du$$

$$= \sin x - \frac{1}{3}u^3 + C$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$

$$\int \sin^3 = \int \sin \cdot \sin^2$$

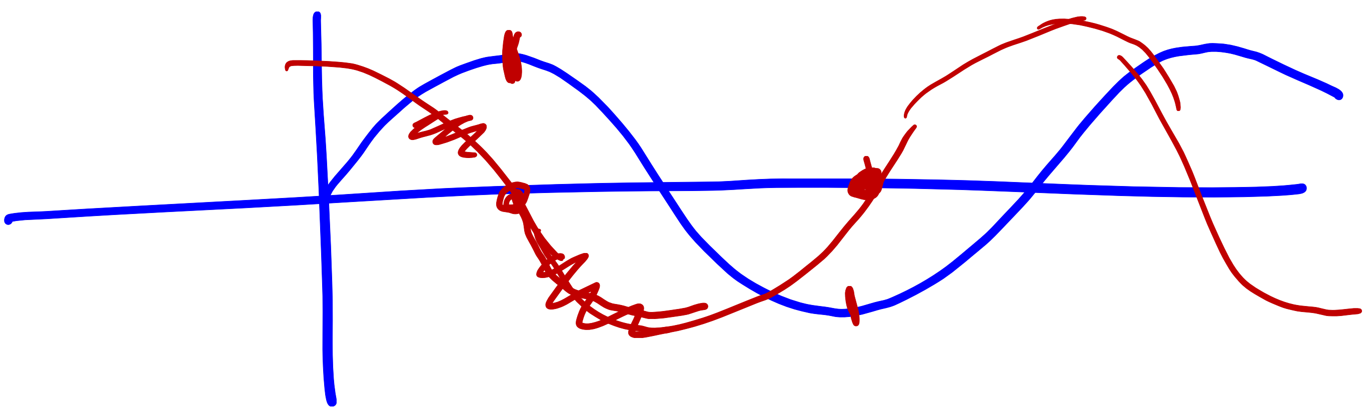
$$= \int \sin(1-\cos^2)$$

$$= \int \sin - \int \cos^2 \sin$$

$u = \cos x$

$\vdots$

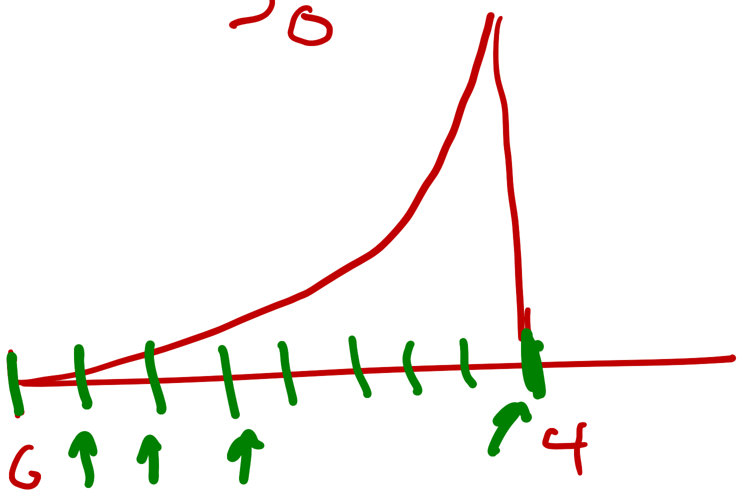
$\sin x$ $\qquad$ $(\sin x)' = \cos x$



$$\int_0^4 x^3\,dx$$

8 steps

Right hand pts



$$\boxed{\sum_{k=1}^{n} \underbrace{\left(\frac{k}{2}\right)^3}_{f(x_k)} \cdot \underbrace{\frac{1}{2}}_{\Delta x}}$$

$$\Delta x = \frac{b-a}{n} = \frac{4}{8} = \frac{1}{2}$$

$$x_k = a + k\Delta x = 0 + k\cdot\frac{1}{2}$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int_1^2 \frac{1}{x\sqrt{1-x^2}} = \frac{1}{1}\sec^{-1}\left(\frac{u}{1}\right) + C$$

$$= \sec^{-1}(x)\Big|_1^2$$

$a = 1$

$$= \sec^{-1}(2) - \sec^{-1}(1)$$

$x = 0$

$$= \cos^{-1}\left(\tfrac{1}{2}\right) - \cos^{-1}(1)$$

$$= \frac{\pi}{3} - 0$$

$$\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx = \sin^{-1}\left(\frac{x}{2}\right) + C$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$$

$a = 2$

$$= \pi/6 - 0$$

$$= \pi/6$$

$\sin^{-1}(\sqrt{3}/2)$

$\sin^{-1}$, $\sec^{-1}$, $\tan^{-1}$

$\sin^{-1}(3/5)$

$$\int_0^1 x^n\,dx = \frac{1}{n+1} x^{n+1} \Big|_0^1$$

0, 1

$$\int \underbrace{x^2}\,\underbrace{\sin(x^3)}\,dx$$

$u = x^3 \quad du = 3x^2\,dx$

$$= \frac{1}{3} \int \sin(x^3)\,(3x\,dx)$$

$$= \frac{1}{3} \int \sin u\,du$$

$$= -\frac{1}{3} \cos u + C = \boxed{-\frac{1}{3}\cos(x^3) + C}$$

1

$$\int \sin x \cos(\cos(x))\,dx$$

$u = \cos x \qquad du = -\sin x\,dx$

$$-\int \cos(\cos(x))\,(-\sin x\,dx)$$

$$= -\int \cos u \; du$$

$$= -\sin(u) + C$$

$$= -\sin(\cos(x)) + C$$

P

$= 20 + 305 -$ ^{2}