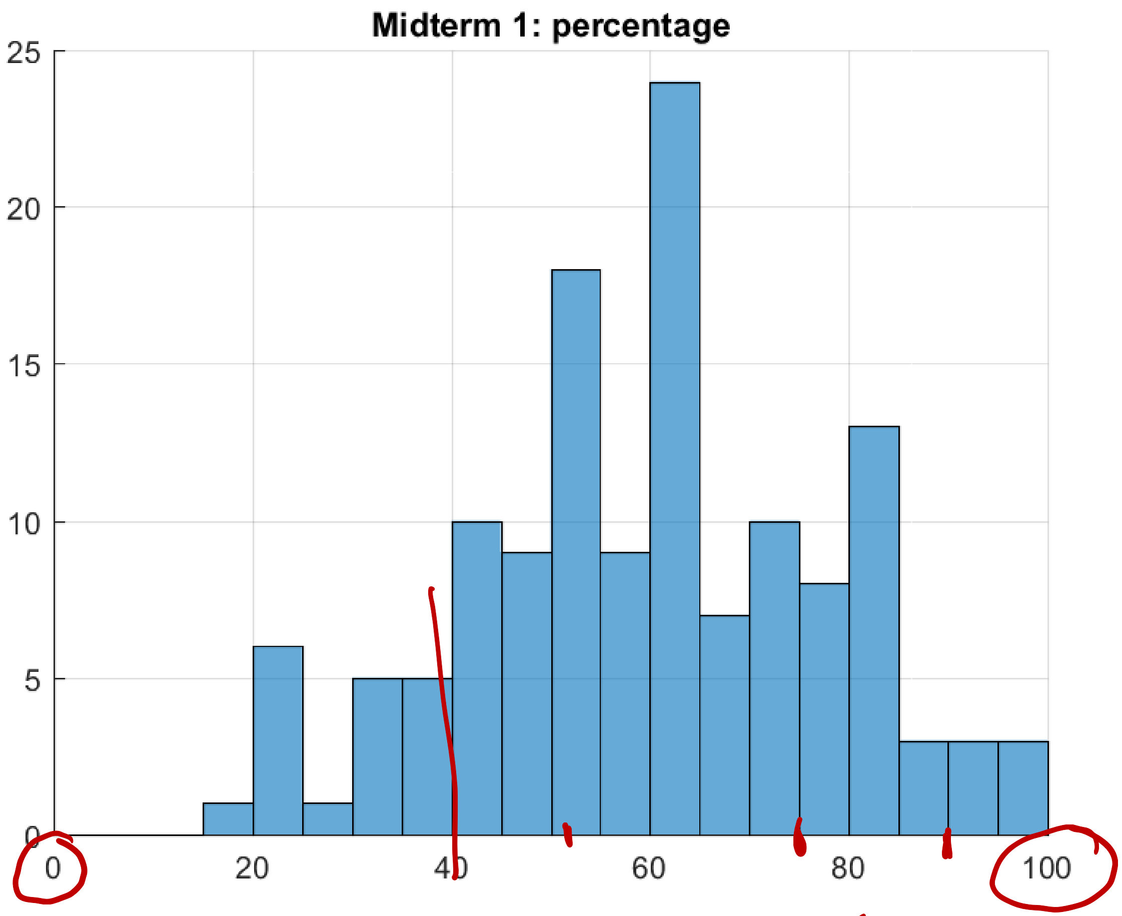
MAT 126.01, Prof. Bishop, Tuesday, Sept. 29, 2020

Discuss Midterm 1

Section 2.2, Volumes by Slicing



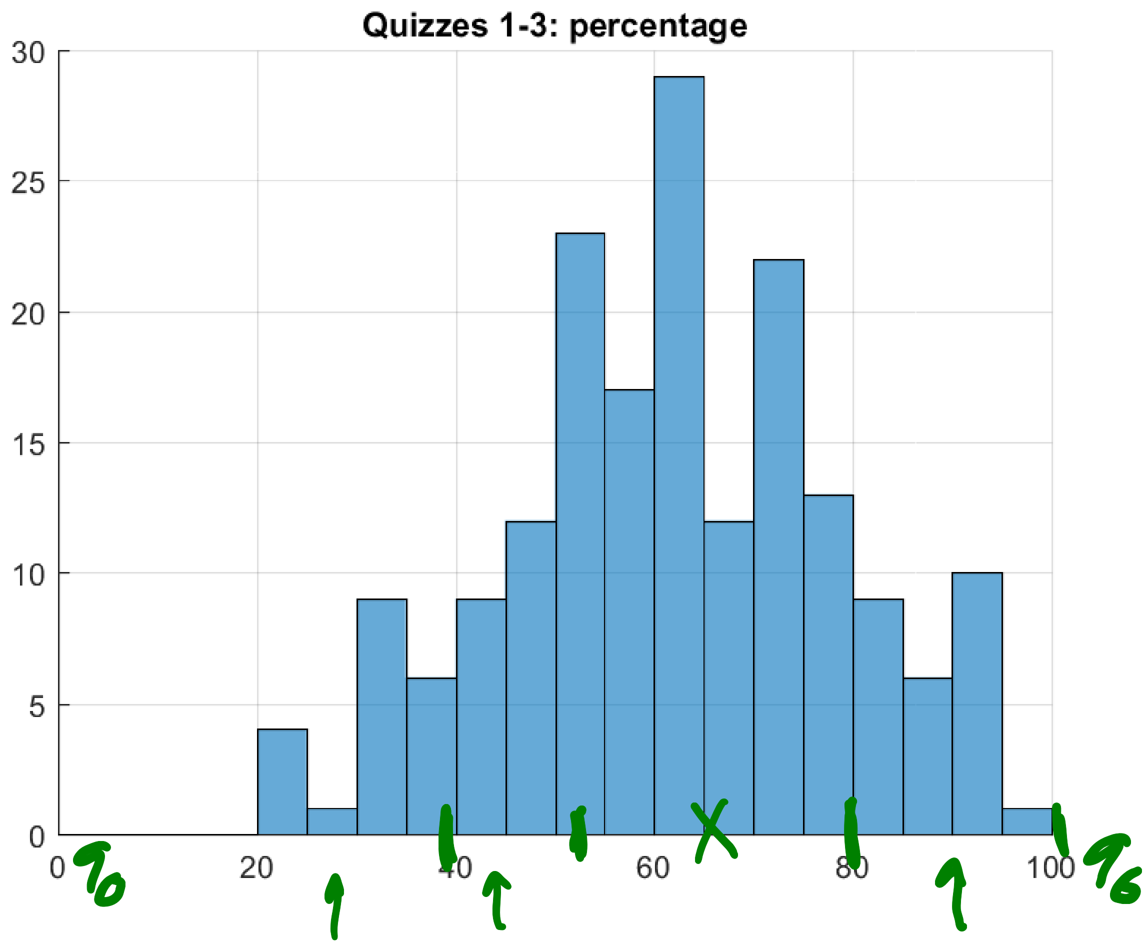
4.0 A

3.0 B

C 12-15

B 16-22

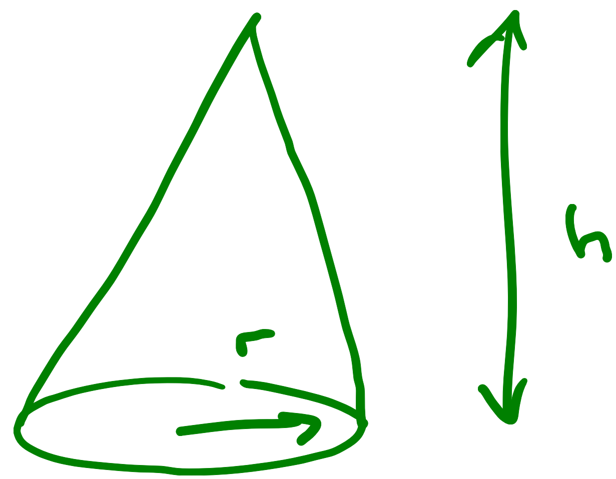
A 23-30


Quizzes 1-3: percentage
30
25
20
15
10
5
0
0
20
40
60
80
100
%
%

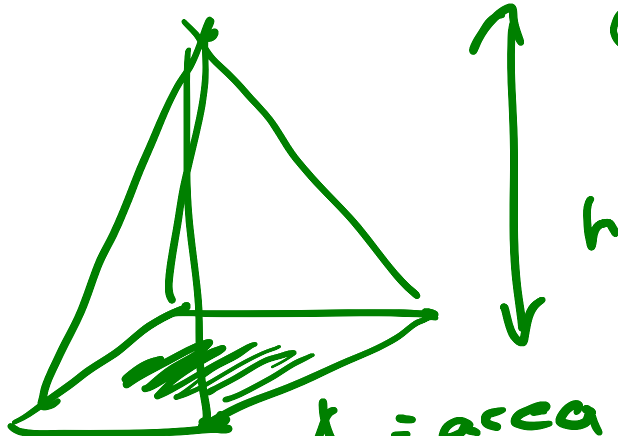
Precalculus volume formulas:

Sphere $V = \frac{4}{3}\pi r^3$

Cone $V = \frac{1}{3}\underbrace{\pi r^2}_{=A} h$

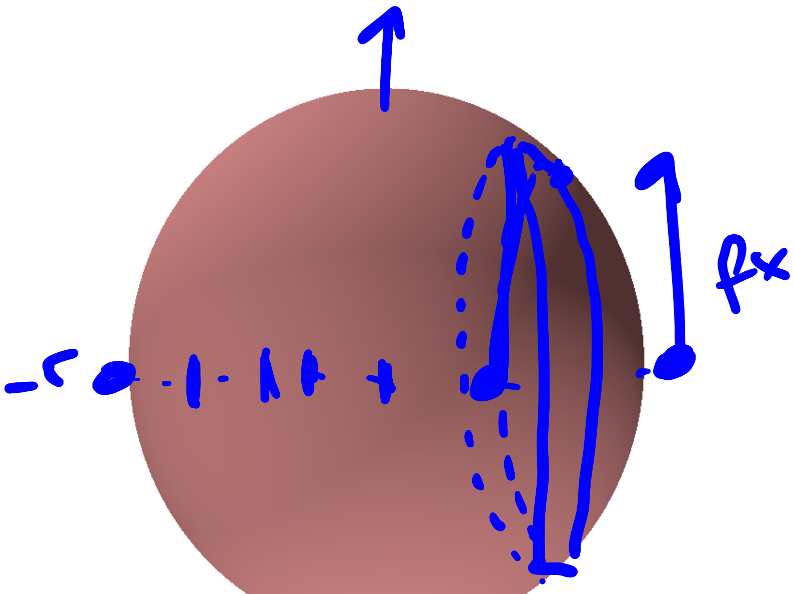


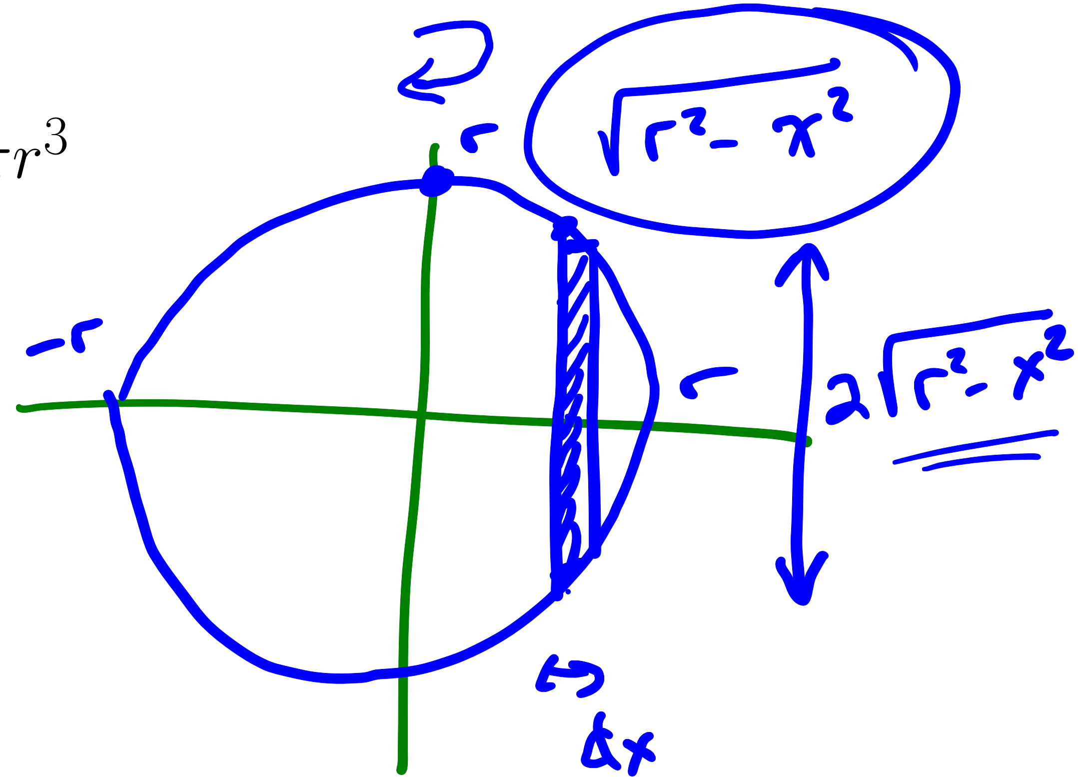
Pyramid $\frac{1}{3}Ah$



A = area of base

Derive volume of sphere $V = \frac{4}{3}\pi r^3$



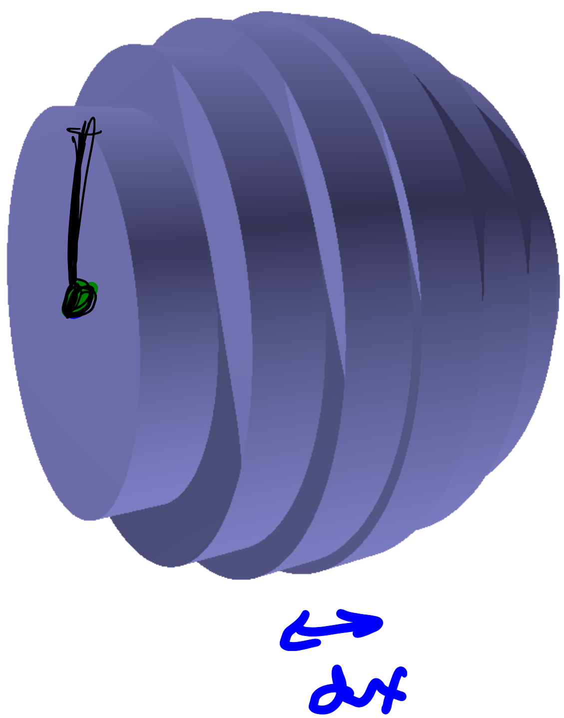


Vol of slice = $dx \cdot$ area of disk

wide $\pi \cdot f(x)^2$

$$V = \int_{-r}^{r} \pi \left(\sqrt{r^2 - x^2}\right)^2 \, dx$$

Derive volume of sphere $\boxed{V = \frac{4}{3}\pi r^3}$



$$V = \int_{-r}^{r} \pi \left(\sqrt{r^2 - x^2}\right)^2 dx$$

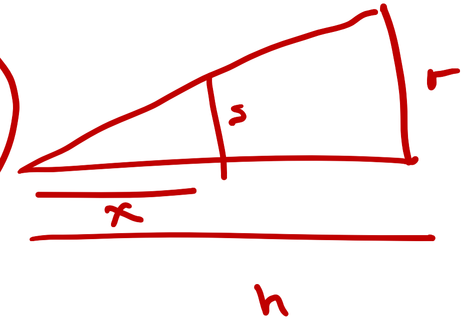
$$= \pi \int_{-r}^{r} (r^2 - x^2)\, dx$$

$$= \pi \left(r^2 x - \frac{1}{3}x^3\right)\Big|_{-r}^{r}$$

$$= \pi \left[\underbrace{\left(r^3 - \frac{1}{3}r^3\right)} + \left(r^3 - \frac{1}{3}r^3\right) \right]$$

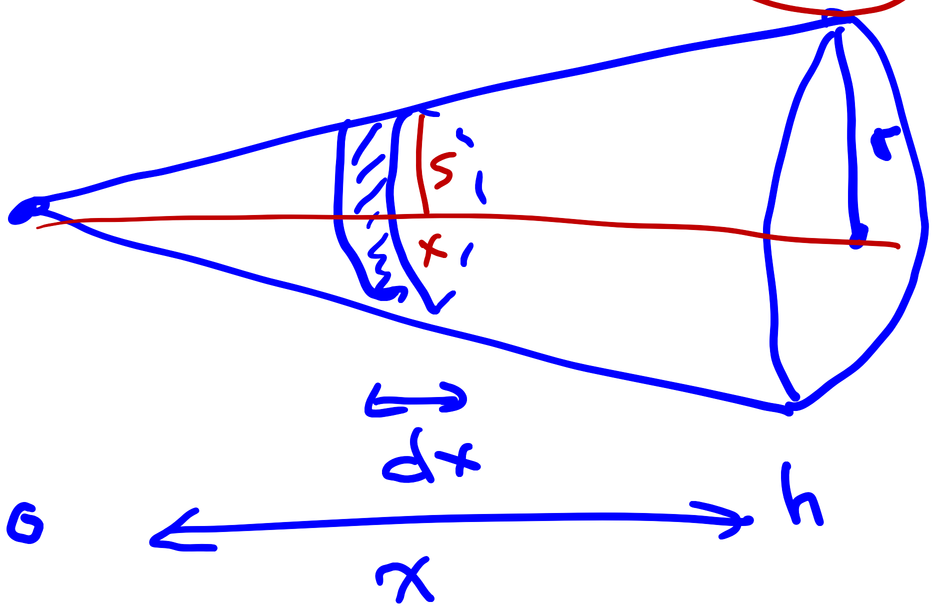
$$= 2\pi \cdot \frac{2}{3} r^3 = \boxed{\frac{4}{3}\pi r^3}$$

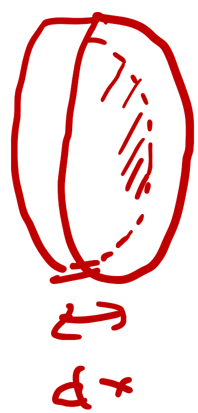
Derive volume of cone $V = \frac{1}{3}\pi r^2 h$



$$\frac{s}{x} = \frac{r}{h}$$

$$s = x \cdot \frac{r}{h}$$





$$s = x\frac{r}{h}$$

Area $\pi s^2 = \pi x^2 \frac{r^2}{h^2}$

$$\int_0^h \pi x^2 \frac{r^2}{h^2}\, dx = \pi \frac{r^2}{h^2} \int_0^h x^2\, dx = \pi \frac{r^2}{h^2} \left(\frac{1}{3}x^3\right)_0^h$$

$$= \pi \frac{r^2}{h^2}\left(\frac{1}{3}h^3 - 0\right)$$

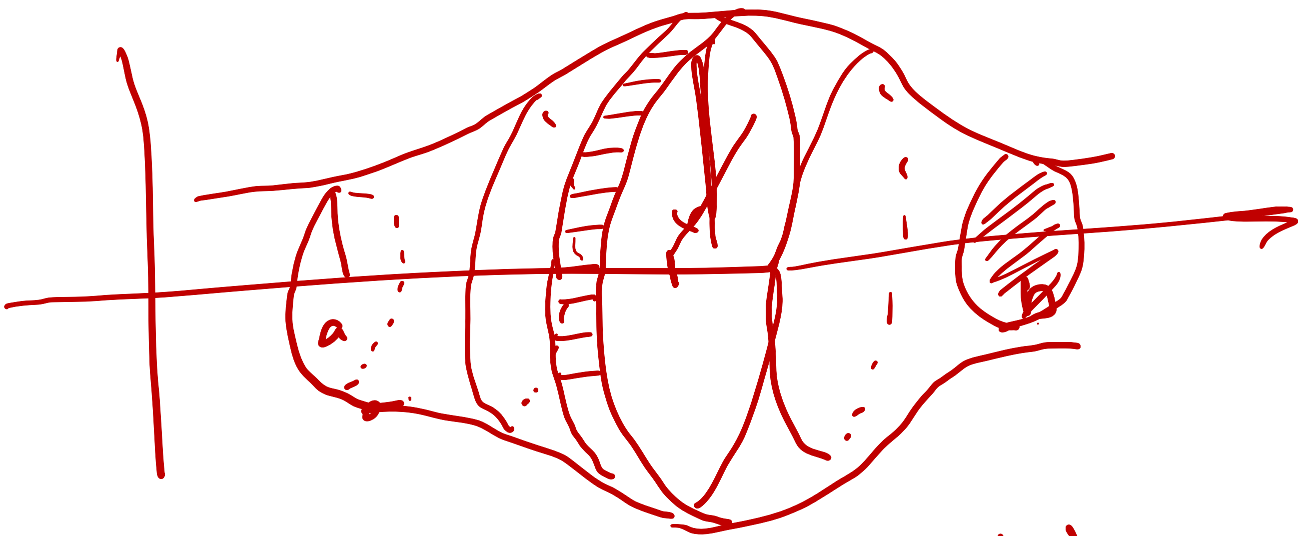
$$= \pi r^2 \frac{1}{3} h$$

If the graph of $f \geq 0$ on $[a, b]$ is revolved around the x-axis, the volume of the solid obtained is

$$\pi \int_a^b f(x)^2 dx.$$

sphere $f = \sqrt{r^2 - x^2}$

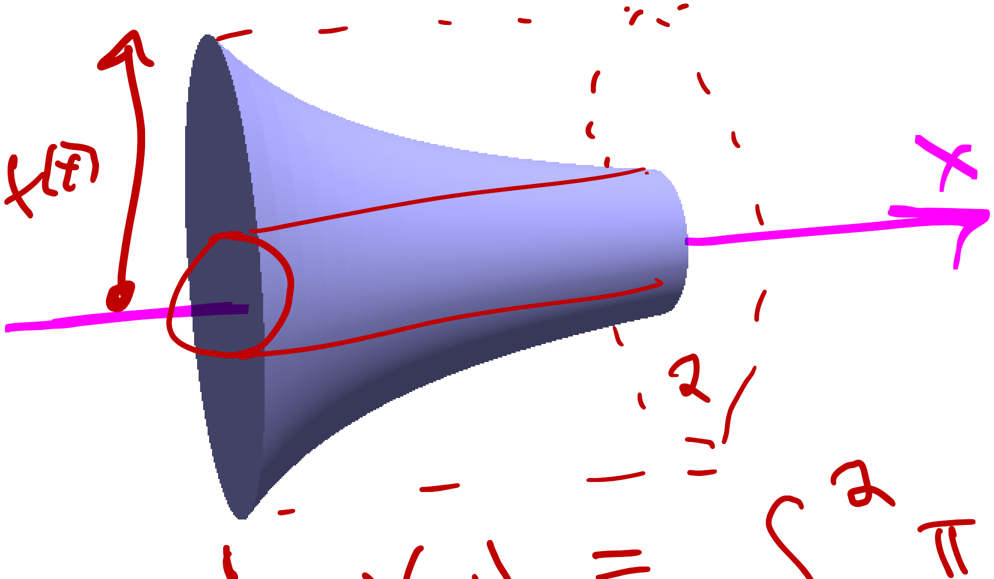
cone $f = x \cdot \frac{r}{h}$

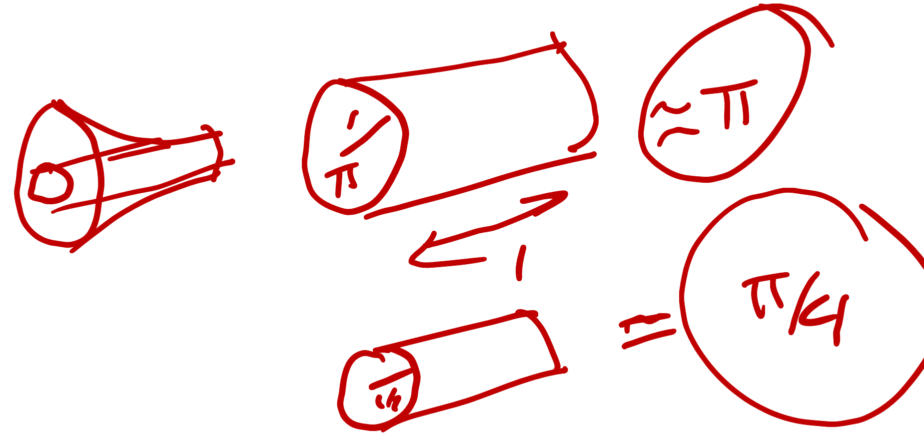


$dx \cdot$ area disk $= dx\, \pi f(x)^2$

Suppose $1/x$ on $[1,2]$ is revolved around x-axis. What is the resulting volume?

$= f(x)$





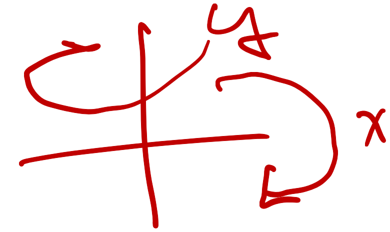
$$\text{Vol} = \int_1^2 \pi f(x)^2\,dx = \int_1^2 \pi\left(\frac{1}{x}\right)^2$$

$$= \int_1^2 \pi x^{-2}\,dx$$

$$= \pi\left(-\frac{1}{x}\Big|_1^2\right) = \pi\left(-\frac{1}{2}-(-1)\right)$$

$$= \underline{\underline{\pi/2}}$$

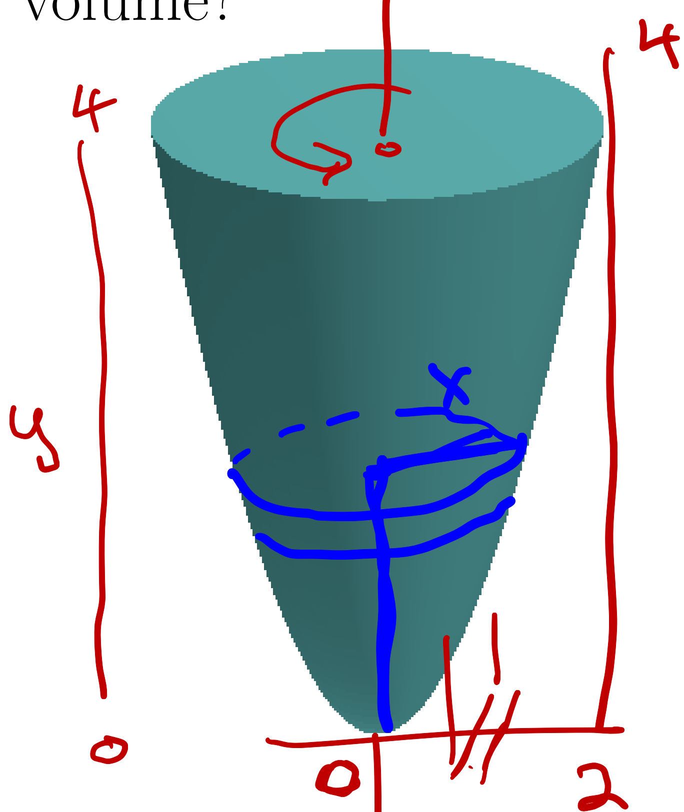
Suppose $\boxed{1/x}$ on $\boxed{[1, \infty)}$ is revolved around x-axis. What is the resulting volume?



$$\boxed{\begin{aligned} &\int_1^\infty \pi f(x)^2\,dx \\ &= \int_1^\infty \pi \left(\frac{1}{x}\right)^2 dx \\ &= \pi \int_1^\infty x^{-2}\,dx \\ &= \pi\left(-\frac{1}{x}\right)\Big|_1^\infty \\ &= \pi\left(0-(-1)\right) = \pi \end{aligned}}$$

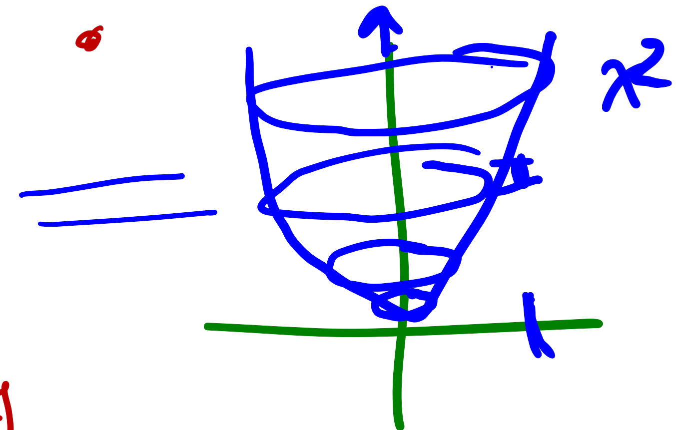
$$\frac{1}{\infty} = 0$$

Suppose x^2 on $[0, 2]$ is revolved around y-axis. What is the resulting volume?



$y = x^2$

$x = \sqrt{y}$



$$\int_0^4 \underbrace{\pi(\sqrt{y})^2}\,\underbrace{dy}$$

$$\boxed{= \pi \int_0^4 y\,dy}$$

$$= \pi\left(\tfrac{1}{2}y^2\right)\Big|_0^4 = \pi(8-0)$$

$$= 8\pi$$

$$\pi(x)^2 dy = \pi(\sqrt{y})^2 dy = \pi y\,dy$$

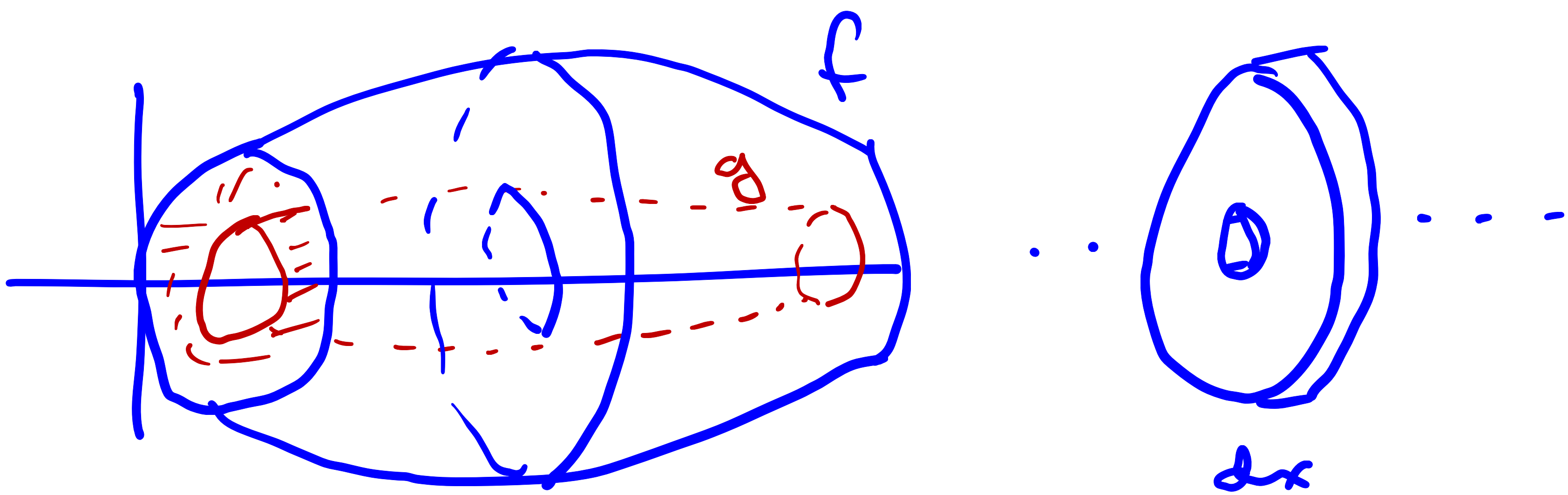
Washer method

If $f \geq g \geq 0$ $[a, b]$ and the region

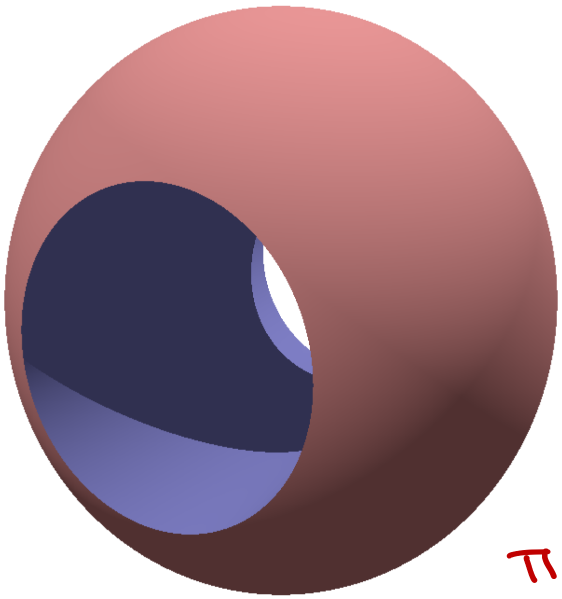
$$\{(x, y) : a \leq x \leq b, g(x) \leq y \leq f(x)\}$$

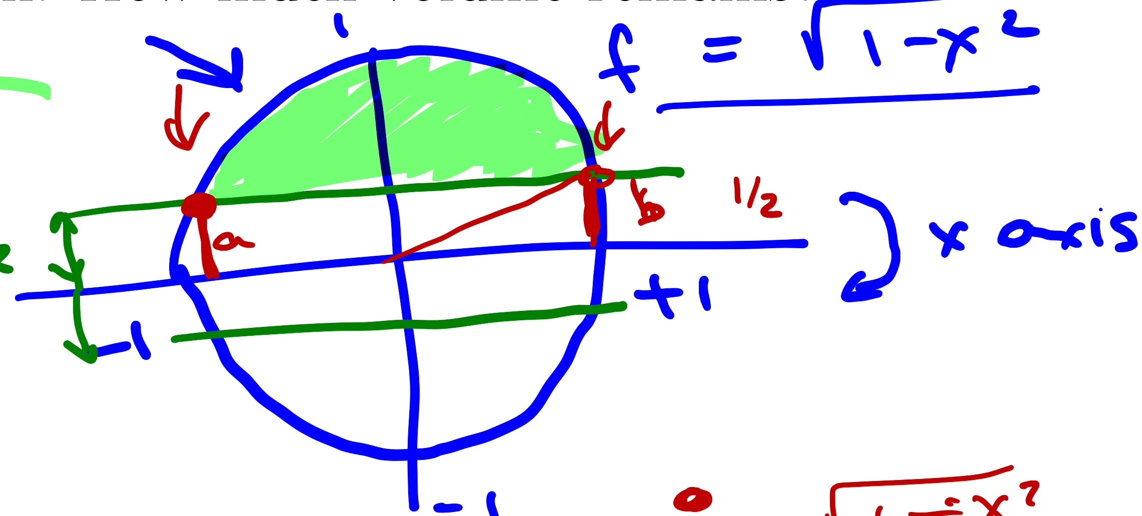
iss revolved around the x-axis, the volume of the solid obtained is

$$\pi \int_a^b (f(x)^2 - g(x)^2)dx. \quad = \pi \int f^2 - \pi \int g^2$$



Suppose a cylinder of radius 1/2 along the x-axis is removed from a sphere of radius 1 centered at the origin. How much volume remains?

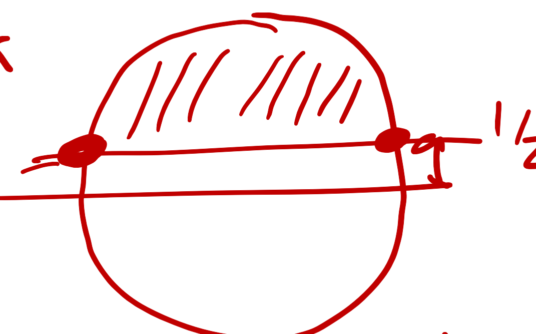




$f = \sqrt{1-x^2}$

$$\pi \int_a^b \left(\sqrt{1-x^2}\right)^2 - \left(\tfrac{1}{2}\right)^2 dx$$

$\sqrt{1-x^2}$



$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$$= \pi \int_{-\sqrt{3}/2}^{\sqrt{3}/2} (1-x^2) - \frac{1}{4}\, dx$$

$$= \pi \int \frac{3}{4} - x^2\, dx$$

$$= \pi \left(\frac{3}{4}x - \frac{1}{3}x^3 \right)\Big|_{-\sqrt{3}/2}^{\sqrt{3}/2}$$

? $\sqrt{1-x^2} = \frac{1}{2}$

$1 - x^2 = \frac{1}{4}$

$\frac{3}{4} = 1 - \frac{1}{4} = x^2$

$\pm\sqrt{3/4} = x$

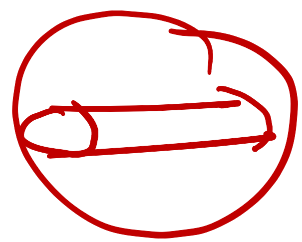
$$= \pi\left(\frac{3}{4}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}\sqrt{3}}{8}\right) - \left(- \quad -\right)$$

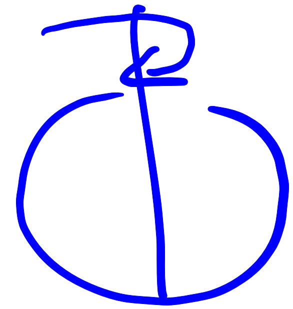
$$= 2\pi\left(\frac{3}{4}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8}\right)$$

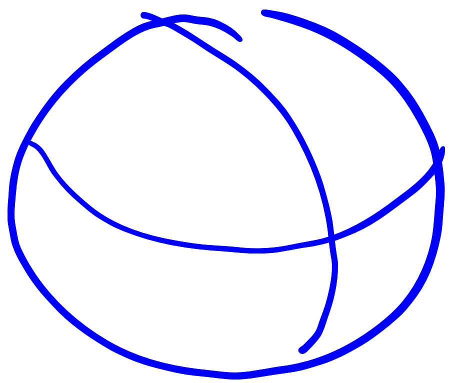
$$= 2\pi\left(\frac{3\sqrt{3}}{8} - \frac{\sqrt{3}}{8}\right)$$

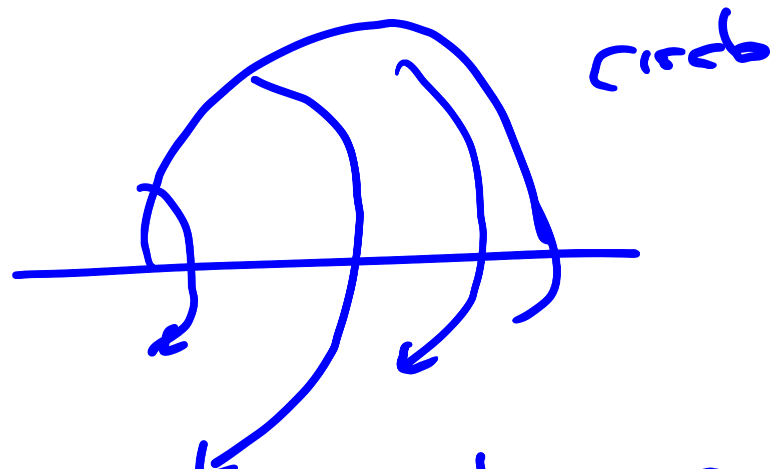
$$= 2\pi\,\frac{2\sqrt{3}}{8}$$

$$= \boxed{\frac{4\sqrt{3}\,\pi}{8}}$$









sphere = circle rotated around x-axis

Office Hours

11:15 - 12:00

a^x exponential e^x 2^x

x^a power x^2 $\sqrt{x}$, $x^{1/2}$

$$\int \frac{dx}{\sqrt{x^2+9}} \ ?$$

—

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{u}{a}\sec^{-1}\left(\frac{u}{a}\right)$$

$$\boxed{\int \frac{dx}{x\ln x\sqrt{(\ln x)^2-1}}} = \ ?$$

$$= \int \frac{1}{\ln x\sqrt{\ln^2 x-1}}\cdot\frac{dx}{x} = \int \frac{1}{u\sqrt{u^2-1}}\,du \qquad a=1$$

$\underline{u = \ln x}$ $\quad du = \frac{1}{x}\,dx$

$$= u\sec^{-1} u + C$$

$$= \ln x \sec^{-1}(\ln x) + C$$

$$\int \frac{\cos(\ln x)}{x}\,dx$$

$u = \ln x \qquad du = \frac{1}{x}\,dx$

$$= \int \cos(u)\,du$$