

APPENDIX A | TABLE OF INTEGRALS

Basic Integrals

$$1. \int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$2. \int \frac{du}{u} = \ln|u| + C$$

$$3. \int e^u du = e^u + C$$

$$4. \int a^u du = \frac{a^u}{\ln a} + C$$

$$5. \int \sin u du = -\cos u + C$$

$$6. \int \cos u du = \sin u + C$$

$$7. \int \sec^2 u du = \tan u + C$$

$$8. \int \csc^2 u du = -\cot u + C$$

$$9. \int \sec u \tan u du = \sec u + C$$

$$10. \int \csc u \cot u du = -\csc u + C$$

$$11. \int \tan u du = \ln|\sec u| + C$$

$$12. \int \cot u du = \ln|\sin u| + C$$

$$13. \int \sec u du = \ln|\sec u + \tan u| + C$$

$$14. \int \csc u du = \ln|\csc u - \cot u| + C$$

$$15. \int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$16. \int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$17. \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$$

Trigonometric Integrals

$$18. \int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

$$19. \int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$20. \int \tan^2 u \, du = \tan u - u + C$$

$$21. \int \cot^2 u \, du = -\cot u - u + C$$

$$22. \int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$$

$$23. \int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$$

$$24. \int \tan^3 u \, du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$$

$$25. \int \cot^3 u \, du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

$$26. \int \sec^3 u \, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$$

$$27. \int \csc^3 u \, du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$$

$$28. \int \sin^n u \, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u \, du$$

$$29. \int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u \, du$$

$$30. \int \tan^n u \, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u \, du$$

$$31. \int \cot^n u \, du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u \, du$$

$$32. \int \sec^n u \, du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u \, du$$

$$33. \int \csc^n u \, du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u \, du$$

$$34. \int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

$$35. \int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

$$36. \int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$37. \int u \sin u \, du = \sin u - u \cos u + C$$

$$38. \int u \cos u \, du = \cos u + u \sin u + C$$

$$39. \int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$$

$$40. \int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$$

$$41. \int \sin^n u \cos^m u \, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du$$

$$= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du$$

Exponential and Logarithmic Integrals

42. $\int u e^{au} du = \frac{1}{a^2}(au - 1)e^{au} + C$
43. $\int u^n e^{au} du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1} e^{au} du$
44. $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C$
45. $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C$
46. $\int \ln u du = u \ln u - u + C$
47. $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$
48. $\int \frac{1}{u \ln u} du = \ln|\ln u| + C$

Hyperbolic Integrals

49. $\int \sinh u du = \cosh u + C$
50. $\int \cosh u du = \sinh u + C$
51. $\int \tanh u du = \ln \cosh u + C$
52. $\int \coth u du = \ln|\sinh u| + C$
53. $\int \operatorname{sech} u du = \tan^{-1}|\sinh u| + C$
54. $\int \operatorname{csch} u du = \ln\left|\tanh \frac{1}{2}u\right| + C$
55. $\int \operatorname{sech}^2 u du = \tanh u + C$
56. $\int \operatorname{csch}^2 u du = -\coth u + C$
57. $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$
58. $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

Inverse Trigonometric Integrals

59. $\int \sin^{-1} u du = u \sin^{-1} u + \sqrt{1-u^2} + C$
60. $\int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1-u^2} + C$
61. $\int \tan^{-1} u du = u \tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$
62. $\int u \sin^{-1} u du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

$$63. \int u \cos^{-1} u \, du = \frac{2u^2 - 1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$64. \int u \tan^{-1} u \, du = \frac{u^2 + 1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$65. \int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$66. \int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$67. \int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} \, du}{1+u^2} \right], n \neq -1$$

Integrals Involving $a^2 + u^2$, $a > 0$

$$68. \int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$69. \int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$70. \int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

$$71. \int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$$

$$72. \int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$$

$$73. \int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} (\sqrt{a^2 + u^2}) - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$$

$$74. \int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$$

$$75. \int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$$

$$76. \int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$$

Integrals Involving $u^2 - a^2$, $a > 0$

$$77. \int \sqrt{u^2 - a^2} \, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln|u + \sqrt{u^2 - a^2}| + C$$

$$78. \int u^2 \sqrt{u^2 - a^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln|u + \sqrt{u^2 - a^2}| + C$$

$$79. \int \frac{\sqrt{u^2 - a^2}}{u} \, du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$$

$$80. \int \frac{\sqrt{u^2 - a^2}}{u^2} \, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln|u + \sqrt{u^2 - a^2}| + C$$

$$81. \int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C$$

$$82. \int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2}\ln|u + \sqrt{u^2 - a^2}| + C$$

$$83. \int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$84. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$$

Integrals Involving $a^2 - u^2$, $a > 0$

$$85. \int \sqrt{a^2 - u^2} du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$86. \int u^2\sqrt{a^2 - u^2} du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$$

$$87. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$$

$$88. \int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u}\sqrt{a^2 - u^2} - \sin^{-1}\frac{u}{a} + C$$

$$89. \int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

$$90. \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$$

$$91. \int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$$

$$92. \int (a^2 - u^2)^{3/2} du = -\frac{u}{8}(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$$

$$93. \int \frac{du}{(a^2 - u^2)^{3/2}} = -\frac{u}{a^2\sqrt{a^2 - u^2}} + C$$

Integrals Involving $2au - u^2$, $a > 0$

$$94. \int \sqrt{2au - u^2} du = \frac{u-a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$95. \int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$96. \int u\sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

$$97. \int \frac{du}{u\sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$$

Integrals Involving $a + bu$, $a \neq 0$

98. $\int \frac{u \, du}{a + bu} = \frac{1}{b^2}(a + bu - a \ln|a + bu|) + C$
99. $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3}[(a + bu)^2 - 4a(a + bu) + 2a^2 \ln|a + bu|] + C$
100. $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$
101. $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
102. $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln|a + bu| + C$
103. $\int \frac{u \, du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
104. $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right) + C$
105. $\int u\sqrt{a + bu} \, du = \frac{2}{15b^2}(3bu - 2a)(a + bu)^{3/2} + C$
106. $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a + bu} + C$
107. $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a + bu} + C$
108. $\int \frac{du}{u\sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \text{ if } a > 0$
 $= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \text{ if } a < 0$
109. $\int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u\sqrt{a + bu}}$
110. $\int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a + bu}}$
111. $\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$
112. $\int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$
113. $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$