MAT 331 Fall 2017, Project 7
Volumes of n-dimensional balls

This project is to use Monte Carlo sampling to verify the formulas for the volume of a unit ball in $\mathbb{R}^n$. The correct formulas and some explanation can be found on the Wikipedia page

`https://en.wikipedia.org/wiki/Volume_of_an_n-ball`

as well as many other sources. e.g., some textbooks on higher dimensional analysis.

(1) Look up the recursive formula the volume of a unit ball in n-space. Use this formula to make a table of numerical values of these volumes for $n = 1, 2, \ldots 10$. (You should already know the volumes of the unit ball in 1 and 2 dimensions, perhaps even for 3).

(2) Look up the formula for the volume of the n-ball in terms of Euler's Gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt.$$

`MATLAB` has a built-in function for this. Use it to compute the volumes a second time. Make a table to compare the values in (1) and (2) are they the same?

(3) Use random sampling to estimate the volume of a unit n-ball by randomly choosing N points in $[-1, 1]^n$ (choose a n-long vector with each coordinate chosen at random in $[-1, 1]$), and then testing if this vector is in the unit ball (i.e., its Euclidean norm is less than 1). Choose a large value of N, say $N = 1,000,000$ and make a table of the estimated volumes. How does it compare to the values you got in part (1)?