MAT 331 Fall 2017, Homework 2
Due in class Thursday, October 5, 2017
Chebyshev polynomials

This homework numerically verifies some propreties of the Chebyshev polynomials. You may wish to look the Wikipedia page on Chebyshev polynomials for further information.

(1) The Chebyshev polynomials on $[-1, 1]$ are defined by the recurrence relation:
$$T_0(x) = 1,$$
$$T_1(x) = x,$$
$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x).$$
Use the recurrence relation to compute $T_1, \ldots, T_{10}$. Give a table of the coefficients of T_{10}. (Hint; see page 67 of Driscoll's book.)

(2) Plot the polynomials $T_0, \ldots, T_5$, all on the same graph. (you may wish to use the command `polyval` to evaluate polynomials in `MATLAB`).

(3) Plot T_{10}. Using the graph find disjoint intervals that contain each root. The use the `MATLAB` command `fzero` to find each root. Make a table of the roots.

(4) Using plots, compare the function $T_n(x)$ to $P_n(x) = \cos(n \arccos(x))$ for $n = 10$. Using $x = [-1 : .0001 : 1]$ and the command `max`, what is the maximum difference of the functions on $[-1, 1]$?

(5) Prove $T_n = P_n$ for all n by showing $P_0 = T_0$ and $P_1 = T_1$ and showing P_n satisfies the same recurrence relation as T_n. (Hint: use the trig identity $\cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y))$.)

(6) Find (by hand) an explicit formula for the roots of P_n in $[-1, 1]$. For $n = 10$ compare these to the roots of T_{10}. Do they agree?