MAT 331 Project, Fall 2018
The area of the Mandelbrot set

The goal of this project is to estimate the area of the Mandelbrot set. There is no known good formula for this area, so we have to estimate it numerically.

The Mandelbrot set is the set of complex numbers c so that the sequence defined by

$$z_0 = 0, \qquad z_{n+1} = z_n^2 + c$$

has absolute value bounded by 2 for all n. `MATLAB` functions for drawing this set were discussed in class and can be downloaded from the class webpage.

The idea is is that if a set M is inside a rectangle R with dimensions $a \times b$, and we choose a point uniformly at random in the rectangle R, then the chance that it lands in M is area(M)/area(R). Thus if we choose N random points in R and K of them land in M we can deduce area$(M) \approx (K/N)$area(R). This is called the "Monte Carlo" method.

(1) State the definition of the Mandelbrot set and use Wikipedia (or other sources) to briefly describe the history of the set and who it is named for.

(2) Write code for drawing a picture of the Mandelbrot set and include a picture with your report. Explain how the code works. You may modify the code I wrote, but you do not need to include the parts which 'blow up' the set.

(3) Write a `MATLAB` function to use a for loop to choose N random points in $R = [-2, 2] \times [-2, 2]$ and for each point run check the definition of the Mandelbrot set for S steps (N and S should be inputs of your function). Output the estimated area.

(4) For $N = 1,000,000$ make a plot of the estimated areas for $S = 10, 100, 1000, 10000$.

(5) How long did the experiments in the previous part take? Using this information, how large could you take N if wanted the program to run for an hour? A day? Do one of these experiments.