MAT 331 Project
Giant components of random graphs

In a large random graph, there tends to be one large connected component, and possibly a number of small ones. In this project, we will explore the size of this "giant" component as a function of the number of vertices V and the number of edges E. For reference see the Wikipedia article

`https://en.wikipedia.org/wiki/Giant_component`

or the paper by Bela Bollobas "The evolution of random graphs" (there is a link on the class webpage).

(1) Write a script that builds a random graph with V vertices and E edges. The input should be V and E and the output is the adjacency graph (it is probably a good idea to use sparse matrices). To define the graph, choose random pairs of vertices and check to see if that edge has already been chosen; if not add it to the graph. Continue until E edges have been chosen. Find the size of the largest connected component of the graph.

(2) The total number of possible edges on V vertices is $N = V(V-1)/2$. Take values of $s > 0$ and set $E = sN$; this is roughly the same as choosing edges with probability s. Write code that takes in a value of s and returns the size of the largest connected component in a random graph with V vertices and $E = sN$ edges.

(3) We are most interested in what happens when the probability of choosing an edge is about $1/V$. Using the code from the previous step Take various values of V (say $100, 500, 1000, 5000, 10000$) take various values of s close to $1/V$, and compute the size of the largest component.

More precisely, take t going from 0 to 4 in steps of .01 and set $s = t/V$. Compute the size of the largest component in the graph with V vertices where edges are chosen at random with probability s. Redo this 20 times for each value of V and s. Plot the results in a single figure. The x-axis should show the values of t and the y-axis the percentage of vertices in the largest component. Each value of V should be a separate curve. Your figure should look something like the figure in

`http://www.math.stonybrook.edu/~bishop/classes/math331.F18/Scripts/Oct4/GiantComponent.eps`

or

`http://www.math.stonybrook.edu/~bishop/classes/math331.F18/Scripts/Oct4/GiantComponent.eps`

(4) For various values of V, take $E = \text{floor}(V/2)$ and compute the size of the largest component (this correesponds to taking $s = 1$ above). What is the prediction given in Bollobas' paper for how large this component is? Is this supported by the results of your experiments? Plot your results as a function of n. Do a log-log plot of your results. Does this plot look linear? If so, what is its approximate slope?