Homework 2. Solutions.

11.4: Problems 10, 18, 28, 36

10. $u \times v = \langle -19, 56, 2\rangle$
18. We can compute the cross product directly to obtain $u \times v = 8\hat{j}$, but observe that any multiple of $\hat{j}$ will be a perpendicular vector to both u, v. The unit vector perpendicular to both is just $\hat{j}$.
28. Let $\vec{AB} = 0.16\langle \cos(60), \sin(60)\rangle$. The torque is given by the cross product $\vec{AB} \times F = 277\hat{\imath}$.
36. The volume is given by the norm of the triple scalar product $|u \cdot (v \times w)| = 72$

11.5: Problems 24, 34, 54, 106

24. The parametric equations are
$x(t) = -2t - 6$
$y(t) = 2t$
$z(t) = 8$
34 If the two lines intersect at a point P, then there exist parameters s_0, t_0 such that

$$\begin{aligned} -3t_0 + 1 &= 3s_0 + 1 \\ 4t_0 + 1 &= 2s_0 + 4 \\ 2t_0 + 4 &= -s_0 + 1 \end{aligned}$$

solving the first two sets of equations we get $t_0 = \frac{1}{2}, s_0 = -\frac{1}{2}$. Since these solutions must also satisfy the last equation, we get

$$2(\frac{1}{2}) + 4 = \frac{1}{2} + 1$$

which is impossible. Thus the lines cannot intersect.

54. Let $\mathfrak{P}_1$ be the plane passing through the two given points and perpendicular to the plane $6x + 7y + 2z = 10$ which will denote by $\mathfrak{P}_2$. The requirement that $\mathfrak{P}_1, \mathfrak{P}_2$ be perpendicular implies that any given vector on the plane $\mathfrak{P}_1$ is parallel to the normal vector to the plane $\mathfrak{P}_2$, which is $n_2 = \langle 6, 7, 2\rangle$. Now, using the two given points, we can find another vector v on the plane $\mathfrak{P}_1$, namely $v = \langle 0, 1, 6\rangle$. Taking the cross product of v and n_2 we obtain a vector normal to $\mathfrak{P}_1$:

$$v \times n_1 = \langle 40, -36, 6\rangle$$

Thus the plane $\mathfrak{P}_1$ is given by the equation

$$40(x - 3) - 36(y - 2) + 6(z - 1) = 0$$

106. To find the angle between two adjacent sides we determine two planes which contain each side respectively and compute the angle between the corresponding normal vectors. The front face is contained in the plane spanned by the vectors $v = \vec{PQ} = \langle 0, 6, 0\rangle$ and $w = \vec{PS} = \langle 1, -1, 8\rangle$ where $P = (6, 0, 0)$, $Q = (6, 6, 0)$ and $S = (7, -1, 8)$. One of the faces adjacent to the front face (the one on the left) is contained on the plane spanned by the vectors $v_1 = \vec{OP} = \langle 6, 0, 0 \rangle$ and $v_2 = w$, where O is the origin. The normal vectors are then $n_1 = v \times w = \langle 48, 0, -6 \rangle$, $n_2 = v_1 \times v_2 = \langle 0, -48, -6 \rangle$ and thus the angle is given by

$$\cos\theta = \frac{n_1 \cdot n_2}{||n_1|| ||n_2||} = \frac{36}{2340}$$

(See sketch at the end of file)

11.6: Problems 5-10, 46

5.c
6.f
7.e
8.d
9.a
10.b

46. The set of points (x, y, z) in $\mathbb{R}^3$ equidistant from $(0, 0, 4)$ and the xy plane must satisfy the equation

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-4)^2} = |z|$$

Rearranging terms we can rewrite this equation as

$$\frac{x^2}{8} + \frac{y^2}{8} = z - 2$$

This is a paraboloid with vertex at $z = 2$.

11.7: Problems 36, 42, 64, 70, 94

36.

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{6}$$
$$\theta = \arctan(\frac{y}{x}) = \arctan(-2)$$
$$\phi = \arccos\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \arccos(\frac{1}{\sqrt{6}})$$

42.

$$x = 7\sin(\frac{\pi}{9})\cos(3\frac{\pi}{4})$$
$$y = 7\sin(\frac{\pi}{9})\sin(3\frac{\pi}{4})$$
$$z = 7\cos(\frac{\pi}{9})$$

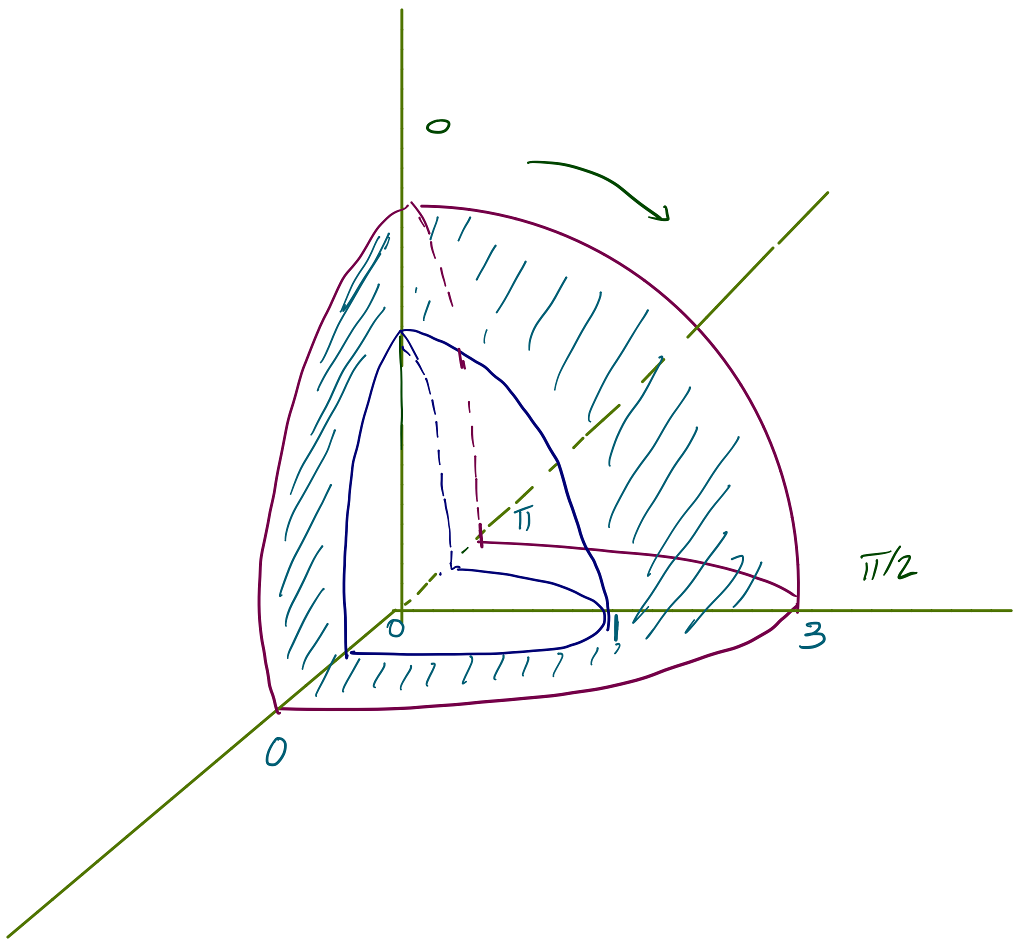
64.

$$\rho = \sqrt{r^2 + z^2} = 5$$
$$\theta = \theta = \frac{\pi}{2}$$
$$\phi = \arccos \frac{z}{\sqrt{r^2 + z^2}} = \arccos \frac{3}{5}$$

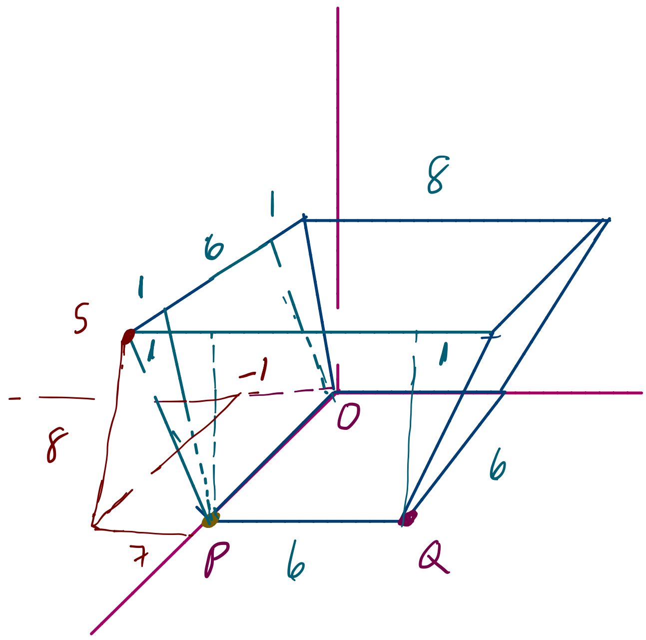
70.

$$r = |\rho \sin(\phi)| = 7\frac{\sqrt{2}}{2}$$
$$\theta = \theta = \frac{\pi}{4}$$
$$z = \rho \cos \phi = -7\frac{\sqrt{2}}{2}$$

94. See sketch below.



Sketch FOR # 106, 11.5



$S = (7, -1, 8)$, $Q = (6, 6, 0)$

$P = (6, 0, 0)$