Homework 6 solutions

§13.6

10. Normalizing $\vec{v}$, we have the direction vector $\vec{u} = \frac{1}{\sqrt{2}}\langle 1, 1\rangle = \langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\rangle$. Now the partial derivatives of h are

$$h_x = -2xe^{-x^2-y^2}, \quad h_y = -2ye^{-x^2-y^2}.$$

Hence the directional derivative $D_{\vec{u}}h(0,0) = h_x(0,0)\cos\frac{\pi}{4} + h_y(0,0)\sin\frac{\pi}{4} = 0$.

18. One can compute the partial derivatives $\frac{\partial z}{\partial x} = -2x\sin(x^2+y^2)$ and $\frac{\partial z}{\partial y} = -2y\sin(x^2+y^2)$. Hence the gradient is

$$\nabla z = \left\langle -2x\sin(x^2+y^2), -2y\sin(x^2+y^2)\right\rangle.$$

Thus, $\nabla z(3,-4) = \langle -6\sin 25, 8\sin 25\rangle$.

24. The gradient is $\nabla f = \langle y+z, x+z, x+y\rangle$, and the unit direction vector of $\vec{v}$ is $\vec{u} = \frac{1}{\sqrt{6}}\langle 2, 1, -1\rangle$. Thus, we have

$$\begin{aligned} D_{\vec{u}}f(1,2,-1) &= \nabla f(1,2,-1)\cdot\vec{u} \\ &= \langle 1,0,3\rangle\cdot\frac{1}{\sqrt{6}}\langle 2,1,-1\rangle = -\frac{1}{\sqrt{6}}. \end{aligned}$$

38. Computing all the partial derivatives of f gives us the gradient

$$\nabla f = \langle e^{yz}, xze^{yz}, xye^{yz}\rangle.$$

Now the maximum value of the directional derivative at $(2,0,-4)$ is just the length of the gradient vector $\nabla f(2,0,-4) = \langle 1,-8,0\rangle$. It is $\sqrt{65}$.

46. The gradient of the given function $f(x,y) = 9x^2 + 4y^2$ is $\nabla f = \langle 18x, 8y\rangle$.

(a) $\nabla f(2,-1) = \langle 36, -8\rangle$.

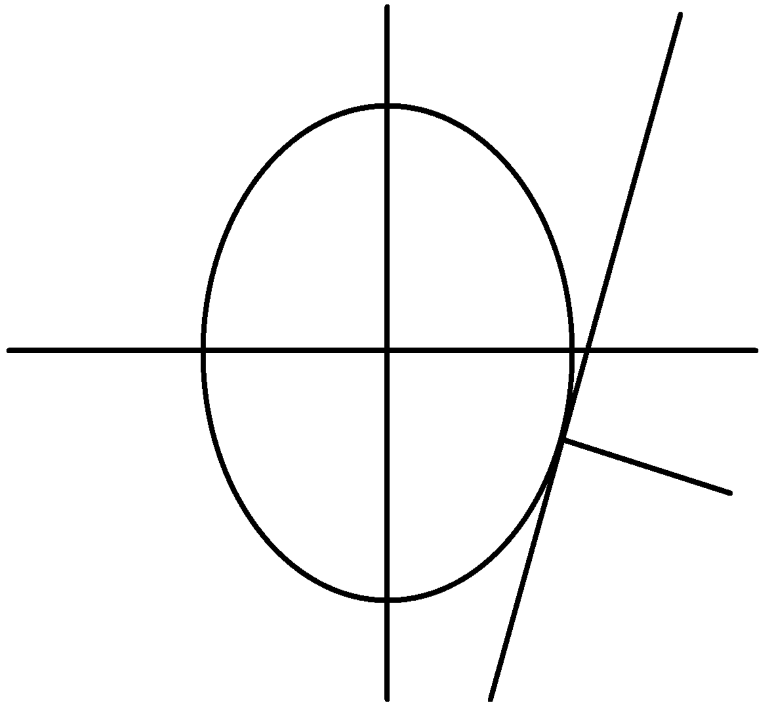
(b) Since the gradient vector $\nabla f(2,-1)$ at $(2,-1)$ is normal to the level curve $f(x,y) = 40$, the desired unit normal vector is the normalization of $\nabla f(2,-1) = \langle 36,-8\rangle$. It is $\frac{1}{\sqrt{85}}\langle 9,-2\rangle$.

(c) The tangent line is perpendicular to the normal vector $\frac{1}{\sqrt{85}}\langle 9,-2\rangle$ and it passes through $(2,-1)$. Hence the defining equation of the tangent line is

$$\frac{1}{\sqrt{85}}\langle 9,-2\rangle\cdot\langle x-2, y+1\rangle = 0.$$

After simplification, it is $9x - 2y = 20$.

(d) The picture is as follows.



Here, the ellipse is the level curve $f(x, y) = 40$, and the tangent line has an equation $9x - 2y = 20$ and it intersects with the ellipse at $(2, -1)$, and the unit normal vector is $\frac{1}{\sqrt{85}}\langle 9, -2\rangle$ with the initial point $(2, -1)$.