MAT 589 Lecture Summaries

Homework. These are the problems from the assigned Problem Set which can be completed using the material from that date's lecture.

Lecture 1. January 28, 2020.

Homework. Problem Set 1 Write up solutions for 5 problems from the problem sets of a classical algebraic geometry course (explained on the webpage).

Proved the Weak and Strong Nullstellensatz. Explained the Ideal-Variety Correspondence. Mentioned theorems of "classical" nature proved using schemes: Castelnuovo's Contractibility Criterion, the Artin-Moishezon Theorem, Mori's proof of Hartshorne's Conjecture, Mori's Cone and Contraction Theorems.

Lecture 2. January 30, 2020.

Continued the Ideal-Variety Correspondence. Defined polynomial mappings of affine algebraic sets. Constructed products in the category of affine algebraic sets. Defined the Zariski topology. Stated the Primary Decomposition Theorem for finitely generated modules over a Noetherian ring.

Fun Problem 1. Find a function between affine algebraic sets such that the graph is an affine algebraic subset of the product of the domain and target, yet the function is not a polynomial morphism. For a challenge, find an example where the domain and target are irreducible and normal.

Lecture 3. February 4, 2020.

Homework. Problem Set 2 Part I: (a), (b), (c), (e); Part II: Problems 2 and 3.

Stated and proved the Principle of Noetherian Induction. Applied this to existence of irreducible decompositions of affine algebraic subsets. Stated the Devissage Lemma and the application to Grothendieck's Generic Freeness result. Stated the corollary: the image of a polynomial mapping is Zariski dense if and only if it contains a Zariski dense open subset of the target. Defined the topological space, Spec, of a commutative unital ring, and proved that Spec of a Noetherian ring is a Noetherian topological space. Defined quasi-affine algebraic sets. Defined regular morphisms. Stated theorem that every regular morphism between affine algebraic sets is a polynomial morphism.

Fun Problem 2. Find an example of a non-Noetherian ring such that Spec is a Noetherian topological space.