Midterm \# 2
Last Name $\quad$ First Name
MAT 127
Last Name
I.D. \#

Lecture\#

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 10 |  |
| 3 | 20 |  |
| 4 | 30 |  |
| 5 | 20 |  |
| Total: | 100 |  |

Stop!
Do Not Open This Exam Booklet
Until You Are Told to Do So!

## Exam Rules:

No Calculators. No Books. No Notes.
Show all your work, explain your reasoning, and cross out anything we should ignore when grading this exam. Also where possible, please always give exact answers (for example, " $\sqrt{5}$ " rather than the decimal approximation " 2.23 ").

You have $\mathbf{9 0}$ minutes to complete this exam.
There are 5 questions, for a total of 100 points. Good luck!

1. The graph below shows a trajectory in the phase plane for a certain predator-prey model. $R$ denotes the number of rabbits and $W$ denotes the number of wolves. Initially (at time $t=0$ ), $R=1000$ and $W=40$.

(a) (10 points) Sketch a rough graph of $R$ as a function of $t=$ time.
(b) (5 points) When the number of rabbits reaches its global maximum, about how many wolves are there?
(c) (5 points) When the number of rabbits reaches its global maximum, is the wolf population increasing or decreasing?
2. (10 points) Give an example of a sequence which is bounded and diverges. Explain why!
3. Determine whether the following sequences are convergent or not. If convergent compute their limits. Show your work!
(a) (10 points)

$$
a_{n}=\frac{\sqrt{n}}{1+\sqrt{n+1}}
$$

(b) (10 points)

$$
a_{n}=\frac{2^{n}}{3^{n+1}}
$$

4. (a) (10 points) Express the number $0 . \overline{32}=0.323232323232 \ldots$ as a ratio of two integers.
(b) (10 points) Evaluate the sum $5+\frac{5}{3}+\frac{5}{9}+\frac{5}{27}+\frac{5}{81}+\cdots$
(c) (10 points) Determine whether the following series converges or not (explain why!)

$$
\sum_{n=1}^{\infty} \frac{5}{3^{n}-n}
$$

Name:
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5. (a) (10 points) Does the series $\sum_{n=2}^{\infty} \frac{\ln (n)}{n}=\frac{\ln (2)}{2}+\frac{\ln (3)}{3}+\frac{\ln (4)}{4}+\cdots$ converge or diverge? Explain why.
(b) (10 points) Does $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{2}}=\frac{1}{2(\ln 2)^{2}}+\frac{1}{3(\ln 3)^{2}}+\frac{1}{4(\ln 4)^{2}}+\cdots$ converge or diverge? Explain why.

