MAT 127 Second Midterm

March 21, 2006

Name: _____ **ID:** _____ **Lect:** _____

Lec 1: Saveleyev, MWF 9:35 Lec 2: Sutherland, TuTh 2:20 Lec 4: Unal, TuTh 5:20

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

There are 5 problems in this exam. Make sure that you have them all.

Do all of your work in this exam booklet, and cross out any work that the grader should ignore. You may use the backs of pages, but indicate what is where if you expect someone to look at it. **Books, calculators, extra papers, and discussions with friends are not permitted.** You may use a time machine to go back and correct your answers, but only if you let me use it afterwards (or is it before?) to go back and change the questions.

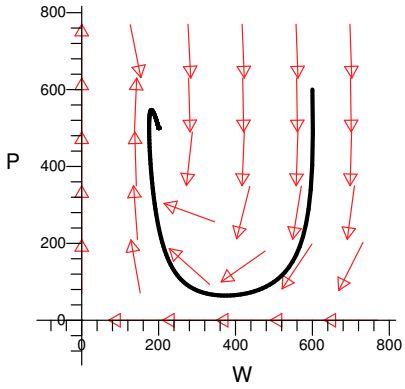
Problems without full justification (ie, "work") will not receive full credit, even for correct answers.

Leave all answers in exact form (that is, do *not* approximate π, square roots, and so on.)

You have 90 minutes to complete this exam.

Name: _____ **Id:** _____

1. [20 points] Two populations, the Pacifists and the Warriors, live near one another. The Pacifists are simple rutabaga farmers: if left to themselves, their population would be well modelled by a logistic growth model. However, the Warriors live nearby, and they survive by making periodic raids on the Pacifists. The two populations are modelled by the predator-prey system below, where t is in years, $W(t)$ is the population of the Warriors after t years, and $P(t)$ is the population of the Pacifists. The phase portrait for this system is shown at right.

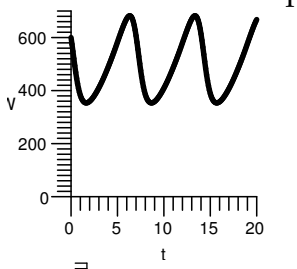


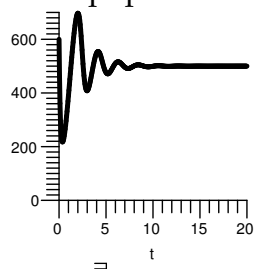
$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{1000}\right) - \frac{PW}{200}$$

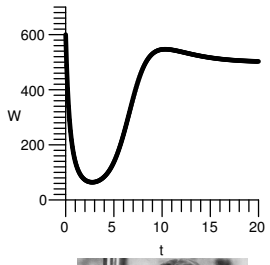
$$\frac{dW}{dt} = -\frac{W}{4} + \frac{PW}{2000}$$

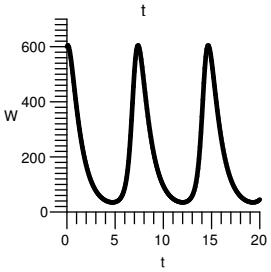
(a) Are there any equilibrium solutions? If so, find **all** of them. If not, write "none", and **justify your answer**.

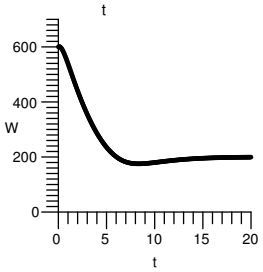
(b) If the populations start out with 600 Pacifists and 600 Warriors, circle the graph below which best represents the population of Warriors.











2. [20 points] For each of the **sequences** below, determine if it converges or diverges. If the sequence converges, state its limit.

(a) $\left\{\dfrac{n^2-4}{n^2+2n}\right\}_{n=1}^{\infty}$

(b) $\left\{\arctan(n!)\right\}_{n=1}^{\infty}$

(c) $\left\{\dfrac{n\sin(n)}{2n-3}\right\}_{n=1}^{\infty}$

(d) The sequence $\{a_n\}_{n=1}^{\infty}$ with $a_0 = 0$ and $a_{n+1} = (a_n)^2 - 2$.

3. [20 points] Find the sums of the convergent series below:

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{3^n} = 1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{2}{n(n+2)} = \frac{2}{3} + \frac{1}{4} + \frac{2}{15} + \dots$ (Hint: partial fractions might be helpful.)

4. [20 points] For each of the series below, decide if it converges or diverges. You do not need to find the value of the sum. **You must justify your answer to receive full credit.**

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2} = \frac{1}{3} + \frac{1}{6} + \frac{1}{11} + \frac{1}{18} + \dots$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n} = \frac{1}{2\ln 2} + \frac{1}{3\ln 3} + \frac{1}{4\ln 4} + \dots$

5. [20 points] A population of rabbits is moved to an island with lots of grass and wild carrots and no predators. Assume the number of rabbits $R(t)$ in each year is well described by a logistic model

$$R'(t) = kR(t)\left(1 - \frac{R(t)}{L}\right)$$

where k and L are appropriate constants.

(a) Suppose that the carrying capacity of the island is 1000 rabbits, and initially there were 250 rabbits. At the end of the first year, there were 400 rabbits. Find an expression for $R(t)$, the number of rabbits after t years.

(b) How many years will it take the population to reach 750 rabbits?