Midterm # 1
MAT 127

_____ , _____
Last Name First Name

_____ _____
I.D.# Lecture#

Run LaTeX again to produce the table

Stop!
Do Not Open This Exam Booklet
Until You Are Told to Do So!

Exam Rules:

No Calculators. No Books. No Notes.

Show Your Work.

You have 90 minutes to complete this exam.

There are ?? questions, for a total of ?? points. Good luck!

1. Find the solution $y(x)$ of the initial value problems

   (a) (10 points)

$$\begin{aligned} \frac{dy}{dx} &= xy \\ y(0) &= 1 \end{aligned}$$

$$y(x) = \underline{\qquad\qquad\qquad\qquad}$$

Answer

   (b) (10 points)

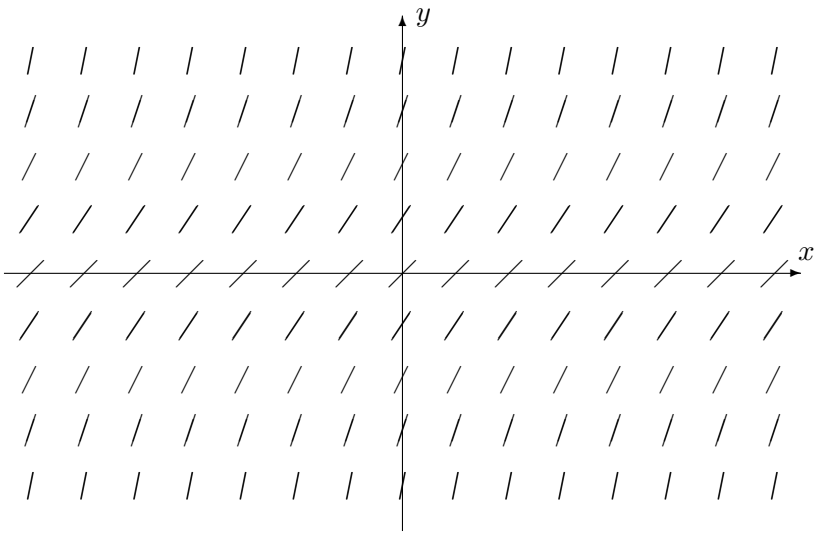
$$\begin{aligned} \frac{dy}{dx} &= xe^x \\ y(0) &= 1 \end{aligned}$$

$$y(x) = \underline{\qquad\qquad\qquad\qquad}$$

Answer

2. The diagram below depicts the *slope field* corresponding to a first order differential equation

$$\frac{dy}{dx} = f(x, y).$$



(a) (10 points) Sketch on the diagram above the solution of the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(0) = 1.$$

(b) (10 points) Does the differential equation have any equilibrium (stationary) solutions?

(c) (10 points) The relevant differential equation is actually one of the equations listed below. Which one? Circle your answer. Give detailed reasons for your answer! (If you need more space use the back of this page.)

(i) $\frac{dy}{dx} = \tan x$

(ii) $\frac{dy}{dx} = \arctan y$

(iii) $\frac{dy}{dx} = 1 + y^2$

(iv) $\frac{dy}{dx} = \frac{\sec^2 x}{1 + y^2}$

(v) $\frac{dy}{dx} = 1 - y^2$

3. Bismuth-210 has a half life of 5 days.

   (a) (10 points) A sample of Bismuth-210 has a mass of 1024 mg. Find a formula for the mass remaining after t days?

   (b) (5 points) Find the mass remaining after 15 days.

   (c) (5 points) When is the mass reduced to 1 mg?

4. (15 points) A tank initially contains 1000 L of brine with 15 kg of dissolved salt. Water is drained from the tank at a rate of 10 liters per minute. Simultaneously, pure water (containing no salt) is added to the tank at a rate of 10 liters per minute. The water in the tank is kept thoroughly mixed, so the salt present is evenly distributed throughout the tank.

   How much salt is left in the tank 100 minutes later?

   Hint: The amount of salt in the tank is a function $y(t)$, where t denotes time (measured in minutes after the start of the experiment). Estimate concentration in terms of $y(t)$ and use this to write down a differential equation that describes the amount $y(t)$ of salt in the tank.

_____ kilograms.

Answer

5. The position $y(t)$ of a weight hanging on a spring is described by the second-order differential equation

$$y'' + 8y' + 15y = 0$$

(a) (8 points) What is the general form of the solution $y(t)$ to this differential equation?

(b) (7 points) If the initial position of the weight is at $y = 0$ and its initial velocity is given by $y'(0) = -4$, what is the position when $t = 1$? You should not approximate e, π, square-roots, or the like.