

Midterm Exam: MAT 342

Instructions: Complete all problems below. You may not use calculators or other aides, including cell phones and books. Show all of your work. **Be sure to write your name and student ID on each page that you hand in.**

1.(15pts) Let $z_1 = -1 + i$ and $z_2 = \sqrt{2}e^{\frac{\pi}{4}i}$.

a) Calculate $\bar{z}_1 z_2$, and write your answer in the form $a + bi$.

b) Calculate $z_1^{1/3}$ and sketch the solution set in the complex plane.

2.(15pts) Determine if the following limits exist or not. If they exist, find the limit:

$$\text{a) } \lim_{z \rightarrow i} \frac{\overline{(z-i)^2}}{(z-i)^2}, \quad \text{b) } \lim_{z \rightarrow e^{\frac{\pi}{4}i}} \frac{z^2 - i}{z^4 + 1}.$$

You must justify all of your work. Simply stating an answer will receive no credit.

- 3.(20pts) a) Sketch the region given by $-\frac{\pi}{6} < \text{Arg}z \leq 0$, $1 < |z| < 2^{1/3}$.
- b) Find the image of the above region under the mapping $w = z^3$.

4.(15pts) If $g(z)$ is a complex analytic function on a domain D , and $\operatorname{Re}(g(z))$ is constant on D , what can you say about $g(z)$? Explain your reasoning in detail.

5.(20pts) Let $z = x+iy$. Find the points where the function $f(z) = (x^2+y^2)+(x-y)i$ is differentiable, and then calculate $f'(z)$ at these points. Is the function analytic at these points? Give full explanations.

6.(15pts) Choose the following branch of the logarithm:

$$\log z = \ln |z| + i\theta, \quad -\frac{5\pi}{3} \leq \theta < \frac{\pi}{3}.$$

Calculate $\log(1+i)^2$ and $2\log(1+i)$ with respect to this branch. Are they equal to each other?