MAT 127: Calculus C, Spring 2015
Solutions to Some HW6 Problems

Below you will find detailed solutions to two problems from HW6. Since they were WebAssign problems, your versions of these problems likely had different numerical coefficients. However, the principles behind the solutions and their structure are as described below.

WebAssign Problem 1

Find the general solution to the differential equation

$$y'' - 13y' + 42y = 0, \qquad y = y(x).$$

The associated quadratic equation is

$$r^2 - 13r + 42 = 0.$$

This gives $(r - 6)(r - 7) = 0$, so the roots are $r = 6, 7$ and the general solution of the differential equation is $\boxed{y(x) = C_1 e^{6x} + C_2 e^{7x}}$

WebAssign Problem 3

Find the general solution to the differential equation

$$2y'' + 3y' = 0, \qquad y = y(x).$$

This is the same as $2y'' + 3y' + 0y = 0$, so the associated quadratic equation is

$$2r^2 + 3r + 0 = 0.$$

This gives $r(2r + 3) = 0$, so the roots are $r = 0, -3/2$ and the general solution of the differential equation is

$$y(x) = C_1 e^{0x} + C_2 e^{-(3/2)x} = \boxed{C_1 + C_2 e^{-3x/2}}$$

Note: This equation can be solved in a different way. Letting $z = y'$, the original equation becomes

$$2z' + 3z = 0, \quad z = z(x) \quad \Longleftrightarrow \quad 2\frac{\mathrm{d}z}{\mathrm{d}x} = -3z \quad \Longleftrightarrow \quad \frac{\mathrm{d}z}{z} = -\frac{3}{2}\mathrm{d}x \quad \Longleftrightarrow \quad \int \frac{\mathrm{d}z}{z} = -\int \frac{3}{2}\mathrm{d}x$$

$$\Longleftrightarrow \quad \ln|z| = -\frac{3}{2}x + C \quad \Longleftrightarrow \quad |z| = \mathrm{e}^{-3x/2+C} = A\mathrm{e}^{-3x/2}$$

$$\Longleftrightarrow \quad y' = z = \pm A\mathrm{e}^{-3x/2} = C\mathrm{e}^{-3x/2}.$$

Integrating the last expression, we obtain

$$y(x) = C \cdot \frac{1}{-3/2}\mathrm{e}^{-3x/2} + C_2 = C_1 \mathrm{e}^{-3x/2} + C_2.$$

This approach works only because the equation involves y'' and y', and not y; so it is really an equation for y'. While this approach is longer, it does not require remembering how to solve second-order linear homogeneous differential equations with constant coefficients (though you should certainly remember this).