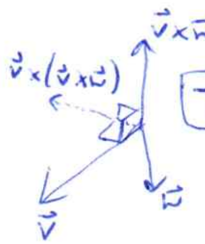
EXAM 1 (SPRING 2020)

1. (a) $\boxed{F}$ (b) $\boxed{T}$ (c) $\boxed{F}$ (d) $\boxed{F}$ (e) $\displaystyle\int_0^{\pi/2} \sqrt{\cos^2 t + \cos^2 t + \cos^2 t}\, dt$

$= \displaystyle\int_0^{\pi/2} \sqrt{3}\, \cos(t)\, dt = \sqrt{3}$ $\boxed{T}$

(f) $\boxed{T}$ (g) $\boxed{T}$ (h) $\vec{v}\times(\vec{v}\times\vec{w})$ $\boxed{T}$



(i) $\boxed{F}$ (j) $y=2$ $\boxed{T}$

2. (a)(i) d (ii) f (iii) e (iv) b (v) a (vi) c

Note: the plots for (e) and (f) seem to be oriented differently from the others. Just focus on shape.

(b) (i) b (ii) d (iii) a (iv) c (v) d (vi) a

3. (a) $\|\vec{PA}\| = \sqrt{(2-1)^2 + (2-1)^2 + (2-1)^2} = \sqrt{1+1+1} = \boxed{\sqrt{3}}$

$\|\vec{AB}\| = \sqrt{(2-2)^2 + (0-2)^2 + (0-2)^2} = \sqrt{4+4} = \boxed{\sqrt{8} = 2\sqrt{2}}$

(b) $\cos(\theta) = \dfrac{\vec{PA}\cdot\vec{PB}}{\|\vec{PA}\|\,\|\vec{PB}\|} = \dfrac{\langle 1,1,1\rangle \cdot \langle 1,-1,-1\rangle}{\sqrt{3}\cdot\sqrt{3}} = \boxed{\dfrac{-1}{3}}$

Note: $\vec{PB} = \langle 2-1, 0-1, 0-1\rangle = \langle 1,-1,-1\rangle$, $\|\vec{PB}\| = \sqrt{1+1+1} = \sqrt{3}$

(c) $\vec{PC} = \langle 0-1, 2-1, 0-1\rangle = \langle -1, +1, -1\rangle$

$$V = \left|\begin{matrix}\vec{PA}\\ \vec{PB}\\ \vec{PC}\end{matrix}\right| = \left|\begin{matrix}1 & 1 & 1\\ 1 & -1 & -1\\ -1 & 1 & -1\end{matrix}\right| = 1\left|\begin{matrix}-1 & -1\\ 1 & -1\end{matrix}\right| - 1\left|\begin{matrix}1 & -1\\ -1 & -1\end{matrix}\right| + 1\left|\begin{matrix}1 & -1\\ -1 & 1\end{matrix}\right|$$

$= 2 + 2 + 2 = \boxed{6}$

Note: This solution uses the "triple scalar product" (more accurately, the determinant) which we haven't covered. There will not be a problem like this on the exam.

4. (a) $\vec{r}\,''(t) = \langle 8t, 0, 8\sqrt{2}\,t^3\rangle$

$\vec{r}\,'(t) = \langle 4t^2, 0, 2\sqrt{2}\,t^4\rangle + \vec{v}_0$ $\quad \vec{r}\,'(0) = \langle 0,0,0\rangle + \vec{v}_0 = \langle 0, 0, -\sqrt{2}\rangle$

$\Rightarrow \vec{v}_0 = \langle 0, 0, -\sqrt{2}\rangle$

$= \langle 4t^2, 0, 2\sqrt{2}\,t^4 - \sqrt{2}\rangle$

$\vec{r}(t) = \langle \frac{4}{3}t^3, 0, \frac{2\sqrt{2}}{5}t^5 - \sqrt{2}\,t\rangle + \vec{r}_0$ $\quad \vec{r}_0 = \langle 2, 0, 3\rangle$

$= \langle 2 + \frac{4}{3}t^3, 0, 3 + \frac{2\sqrt{2}}{5}t^5 - \sqrt{2}\,t\rangle$ $\quad \vec{r}(1) = \langle 2 + \frac{4}{3}, 0, 3 + \frac{2\sqrt{2}}{5} - \sqrt{2}\rangle$

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(b) $\int_0^1 \|\vec{r}\,'(t)\|\,dt = \int_0^1 \sqrt{16t^4 + (2\sqrt{2}t^4 - \sqrt{2})^2}\,dt$

$= \int_0^1 \sqrt{16t^4 + 8t^8 - 8t^4 + 2}\,dt$

$= \int_0^1 \sqrt{8t^8 + 8t^4 + 2}\,dt = \int_0^1 \sqrt{(\sqrt{8}t^4 + \sqrt{2})^2}\,dt$

$= \int_0^1 \sqrt{8}\,t^4 + \sqrt{2}\,dt = \left(\frac{\sqrt{8}}{5}t^5 + \sqrt{2}\,t\right)\Big|_0^1 = \frac{\sqrt{8}}{5} + \sqrt{2} = \frac{2\sqrt{2}}{5} + \sqrt{2}$

$= \boxed{\frac{7\sqrt{2}}{5}}$

5. (a) $y = t$

$x = y^3 - (y^2 - 2) = t^3 - t^2 + 2$ $\qquad \vec{r}(t) = \langle t^3 - t^2 + 2,\ t,\ t^2 - 2\rangle$

$z = t^2 - 2$

(b) Take $t = 1$. $\qquad \vec{r}\,'(t) = \langle 3t^2 - 2t,\ 1,\ 2t\rangle$

$\vec{r}\,'(1) = \langle 1, 1, 2\rangle$

$x = 2 + t, \quad y = 1 + t, \quad z = -1 + 2t$

(c) $\vec{r}\,''(t) = \langle 6t - 2, 0, 2\rangle$ $\qquad \vec{r}\,'(1) \times \vec{r}\,''(1) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 4 & 0 & 2 \end{vmatrix}$

$\vec{r}\,'(1) = \langle 4, 0, 2\rangle$

$= 2\vec{i} - (2 - 8)\vec{j} + (-4)\vec{k}$

$= \langle 2, 6, -4\rangle$

$\|\vec{r}\,'(1) \times \vec{r}\,''(1)\| = \sqrt{4 + 36 + 16} = \sqrt{56}$

$K = \frac{\|\vec{r}\,'(1) \times \vec{r}\,''(1)\|}{\|\vec{r}\,'(1)\|^3} = \frac{\sqrt{56}}{(\sqrt{1+1+4})^3} = \frac{\sqrt{56}}{\sqrt{6^3}} = \sqrt{\frac{56}{6^3}} = \sqrt{\frac{56}{216}} = \sqrt{\frac{7}{27}}$

<u>Note</u>: The formula

$K = \frac{\|T'(t)\|}{\|\vec{r}\,'(t)\|}$ seems impractical for this problem.

(d) $\vec{v} = \langle 1, 1, 2\rangle$ $\qquad D = \frac{\|\vec{OA} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{9 + 25 + 1}}{\sqrt{1 + 1 + 4}} = \boxed{\sqrt{\frac{35}{6}}}$

$\vec{OA} = \langle 2, 1, -1\rangle$

$\vec{OA} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -1 \\ 1 & 1 & 2 \end{vmatrix} = (2+1)\vec{i} - (4+1)\vec{j} + (2-1)\vec{k}$

$= 3\vec{i} - 5\vec{j} + \vec{k}$