EXAM 5 (FALL 2019)

1. $$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = -\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$$

Thus $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ as claimed.

2. $$\frac{\partial f}{\partial x} = 3(x^2+y)^2(2x) \qquad \frac{\partial f}{\partial y} = 3(x^2+y)^2(1)$$

$f(1,0) = (1+0)^3 = 1$ $\quad \frac{\partial f}{\partial x}(1,0) = 3(1)(2) = 6$ $\quad \frac{\partial f}{\partial y}(1,0) = 3(1)(1) = 3$

Linear approximation is $z = L(x,y) = 6(x-1) + 3y + 1$

3. $F(x,y,z) = e^{xz} + yz - 2 = 0$

$\nabla F = \langle e^{xz} z,\ z,\ e^{xz} x + y \rangle$

$\nabla F(0,1,1) = \langle 1, 1, 1 \rangle$

$$1(x-0) + 1(y-1) + 1(z-1) = 0$$

$x + y + z - 2 = 0$