# MIDTERM 1 FORMULAS

Notation: $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ are vectors; $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$; $\mathbf{r}(t)$ is a vector-valued function defined on $[a, b]$; $f(x, y)$ a function defined on a planar region

Distance $d$ between points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$

Projection of $\mathbf{u}$ onto $\mathbf{v}$: $\text{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v}$

Cross product: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - v_2u_3)\mathbf{i} - (u_1v_3 - v_1u_3)\mathbf{j} + (u_1v_2 - v_1u_2)\mathbf{k}$

Identities: $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\,\|\mathbf{v}\|\,\cos(\theta)$ and $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|\sin(\theta)$

Cylindrical coordinates: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = z$.

Spherical coordinates: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$.

Unit tangent vector: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$, if $\mathbf{r}'(t) \neq \mathbf{0}$

Principal unit normal vector: $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$, if $\mathbf{T}'(t) \neq \mathbf{0}$

Tangential component of acceleration: $a_{\mathbf{T}} = \dfrac{d}{dt}\|\mathbf{r}'(t)\| = \dfrac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{\|\mathbf{r}'(t)\|}$

Normal component of acceleration: $a_{\mathbf{N}} = \|\mathbf{r}'(t)\|\,\|\mathbf{T}'(t)\| = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}$

Arc length: $s = \displaystyle\int_a^b \|\mathbf{r}'(t)\|\,dt$

Curvature: $K = \left\|\dfrac{d\mathbf{T}}{ds}\right\| = \dfrac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \dfrac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

Gradient: $\nabla f(x, y) = \langle f_x(x, y), f_y(x, y)\rangle$