Review: Green's theorem

Green's theorem

Let R be a simply connected region with piecewise smooth boundary C, oriented counterclockwise. Let $\mathbf{F} = \langle M, N \rangle$, where M, N have continuous first partial derivatives. Let $\mathbf{N}$ be the outward pointing normal vector. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \mathbf{F} \cdot \mathbf{T}\, ds = \iint_R \operatorname{curl} \mathbf{F}\, dA$$

and

$$\int_C \mathbf{F} \cdot \mathbf{N}\, ds = \iint_R \operatorname{div} \mathbf{F}\, dA.$$

Stokes's theorem: introduction

- Stokes's Theorem generalizes the circulation (curl) form of Green's Theorem.
- The main difference is that the region R is replaced by a surface in space.

Stokes's theorem

Let S be an oriented surface in $\mathbb{R}^3$ with unit normal vector $\mathbf{N}$ bounded by a simple closed curve C. Then

$$\int_C \mathbf{F}\, d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{N}\, dS.$$