Stony Brook University
Department of Mathematics

MAT203– Midterm #1
Calculus III

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26 Feb. 2020

Name: _____
Student ID Number: _____

This exam contains 11 pages (including this cover page) and 5 questions. Total of points is 60. Good luck!

Distribution of Marks

Question	Points	Score
1	10	
2	12	
3	8	
4	10	
5	20	
Total:	60	

1. True or False:

(a) (1 point) The equation $x + z = 1$ defines a line in space.
◯ True ◯ False

(b) (1 point) The velocity of $\vec{r}(t) =< t, 2t, 3t >$ at $t = 0$ is the same as $t = 1$.
◯ True ◯ False

(c) (1 point) The vector $< 3, -1, 2 >$ is perpendicular to the line $< 3 + t, -1 + 2t, 2 + 3t >$.
◯ True ◯ False

(d) (1 point) The vector $< 3, -1, 2 >$ is parallel to the line $< 3 + t, -1 + 2t, 2 + 3t >$.
◯ True ◯ False

(e) (1 point) The arclength of $\vec{r}(t) =< \sin t, \sin t, \sin t >$ where t from 0 to $\pi/2$ is $\sqrt{3}$.
◯ True ◯ False

(f) (1 point) If the dot product of two unit vectors $\vec{v}$ and $\vec{u}$ is -1, then $\vec{v} = -\vec{w}$.
◯ True ◯ False

(g) (1 point) If two planes do not intersect, then their normal vectors are parallel..
◯ True ◯ False

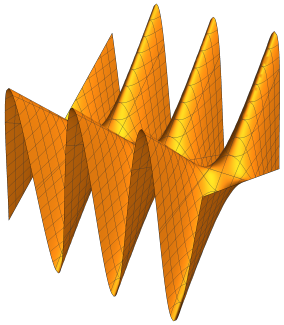
(h) (1 point) The vector $\vec{v} \times \vec{w}$ is parallel to $\vec{v} \times (\vec{v} \times (\vec{v} \times \vec{w}))$.
◯ True ◯ False

(i) (1 point) There are two unit vectors $\vec{v}$ and $\vec{w}$ with $\|\vec{v} \times \vec{w}\| = 2$.
◯ True ◯ False

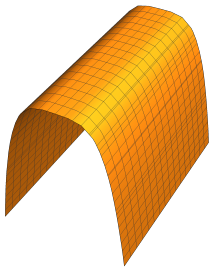
(j) (1 point) The equation $\rho \sin \varphi \sin \theta = 2$ in spherical coordinates defines a plane.
◯ True ◯ False

2. (a) (6 points) Match the equations with the surfaces.

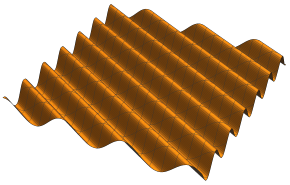
(i) $5x^2 + y^2 - z^2 = 1$ _____
(ii) $5x^2 + y^2 - z^2 = 0$ _____
(iii) $5x^2 + y^2 - z^2 = -1$ _____
(iv) $x^4 + z = 1$ _____
(v) $z = y \sin x$ _____
(vi) $z = \sin(x + 2y)$ _____



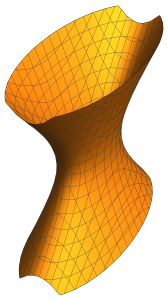
(a)



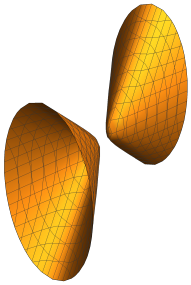
(b)



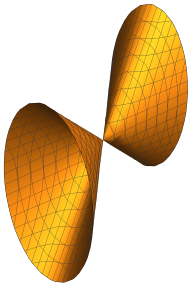
(c)



(d)



(e)



(f)

(b) (6 points) Match the vector valued functions with the curve.

(i) $\vec{r}(t) =< t, t\sin t, 1-t >$ _____

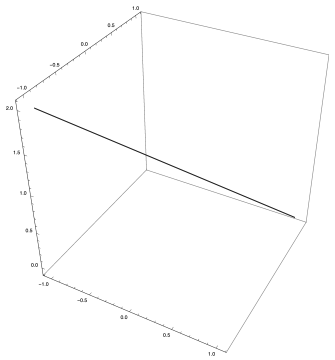
(ii) $\vec{r}(t) =< e^{t^2}, 10\sin e^{t^2}, 1-e^{t^2} >$ _____

(iii) $\vec{r}(t) =< t, t, 1-t >$ _____

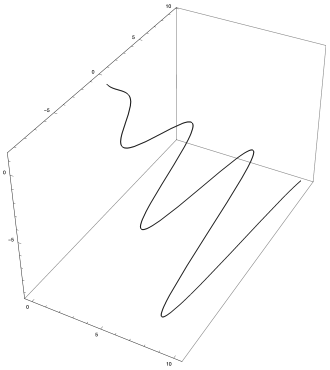
(iv) $\vec{r}(t) =< 2\cos t, \cos(4t), 3\sin t >$ _____

(v) $\vec{r}(t) =< t, 10\sin t, 1-t >$ _____

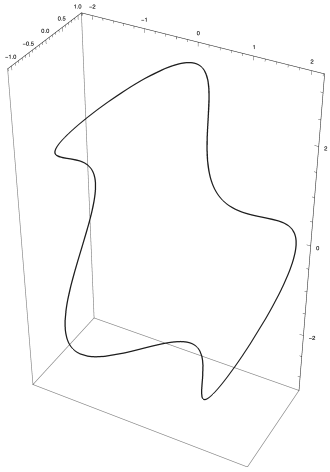
(vi) $\vec{r}(t) =< \sin t, \sin t, 1-\sin t >$ _____



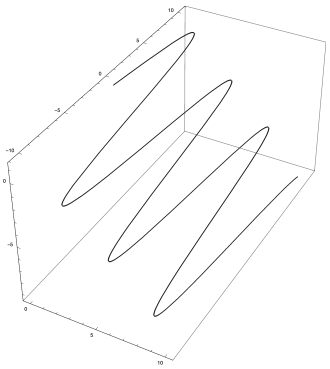
(a)



(b)



(c)



(d)

3. Methane CH_4 is the number two greenhouse gas emitted by human activity in the US. The four hydrogen atoms of methane are located at the vertices $A = (2, 2, 2)$ $B = (2, 0, 0)$, $C = (0, 2, 0)$ and $D = (0, 0, 2)$ and form a regular tetrahedron, while the carbon atom is located at the center $P = (1, 1, 1)$

   (a) (2 points) Find the bond distance of $\|\overrightarrow{PA}\|$ and $\|\overrightarrow{AB}\|$.

   (b) (2 points) Find the cosine of the bond angle between $\overrightarrow{PA}$ and $\overrightarrow{PB}$.

(c) (4 points) What is the volume of the parallelepiped spanned by $\overrightarrow{PA}$, $\overrightarrow{PB}$ and $\overrightarrow{PC}$.

4. SpaceX, is a private aerospace manufacturer and space transportation services company. In February 2020, SpaceX announced plans to fly private citizens into orbit on one of its rocket. Assume the rocket experiences an acceleration

$$\vec{r}''(t) =< 8t, 0, 8\sqrt{2}t^3 >$$

starts at one of its launching facility at Cape Canaveral $\vec{r}(0) =< 2, 0, 3 >$ with initial velocity $\vec{r}'(0) =< 0, 0, -\sqrt{2} >$.

(a) (5 points) Find the location of the rocket at time $t = 1$.

(b) (5 points) Find the total distance (arclength) traveled by the rocket from $t = 0$ to $t = 1$.

5. Consider the curve C of the intersection of the surfaces $x = y^3 - z$ and $z = y^2 - 2$ and the point $A = (2, 1, -1)$ on the curve.

   (a) (5 points) Find a parameterization $\vec{r}(t)$ of the curve C.
   Hing: You can start by setting $y = t$.

   (b) (5 points) Find a parametric equation for the tangent line of C at A.

(c) (5 points) Find the curvature of the curve C at the point A.

(d) (5 points) Find the distance between the origin $(0, 0, 0)$ to the tangent line L.

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.