Math 203 - Fall 2018
Second Examination
Thursday, Nov 8, 2018

Instructor: **Dror Varolin**

This examination contains 10 pages, including this title page, and 4 sheets of scratch paper followed by a formula page at the end. You can tear out the scrap paper and formula sheet if you like.

Read all the questions carefully before starting the exam.

Use of Calculators is not permitted!

Place your final answers in the squares provided!!

Show all your work!!!

Good Luck!!!!

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. Consider the function $z = f(x, y)$ is defined implicitly by the equation

$$z^2(1 + \sin(xy)) + e^{2z-y} = 2$$

and the condition $f(0, 2) = 1$. Compute $\frac{\partial f}{\partial x}(0, 2)$.

$$\boxed{\frac{\partial f}{\partial x}(0, 2) =}$$

2. Consider the function

$$f(x,y) = e^{-(x^2+y^2)}$$

and the unit vector

$$\mathbf{u}_o = \left\langle \tfrac{-1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}} \right\rangle .$$

(a) Calculate the directional derivative $D_{\mathbf{u}_o} f(x,y)$ of f at the point (x,y) along the direction $\mathbf{u}_o$.

$$\boxed{D_{\mathbf{u}_o} f(x,y) =}$$

(b) Find the maximum value of $D_{\mathbf{u}_o} f(x,y)$ among all points (x,y) in the plane, and the point where this maximum occurs.

$$\boxed{\max D_{\mathbf{u}_o} f = \qquad \text{Occurs at } (\quad , \quad)}$$

3. Find the point (x_o, y_o) whose y-coordinate has the largest possible value among all points on (x, y) lying on the curve

$$(2x - y)^2 + 2(x + 3y)^2 = 3.$$

$$\boxed{(x_o, y_o) = (\quad , \quad)}$$

4. Compute the iterated integral

$$\int_0^2 \int_y^{\sqrt{8-y^2}} \int_0^{\sqrt{8-x^2-y^2}} \frac{2\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} dzdxdy.$$

Integral =

Scratch paper

Scratch paper

Scratch paper

Scratch paper

Standard Formulas

- Integration-by-parts formula:

$$\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx.$$

- Trigonometric summation formula:

$$\begin{aligned}\cos(a+b) &= (\cos a)(\cos b) - (\sin a)(\sin b)\\ \sin(a+b) &= (\cos a)(\sin b) + (\sin a)(\cos b)\end{aligned}$$

- Polar to Cartesian coordinate transformation

$$\begin{aligned}x &= r\cos\theta\\ y &= r\sin\theta\\ dA &= rdrd\theta\end{aligned}$$

- Cylindrical to Cartesian coordinate transformation

$$\begin{aligned}x &= r\cos\theta\\ y &= r\sin\theta\\ z &= z\\ dV &= rdr\ d\theta\ dz\end{aligned}$$

- Spherical to Cartesian coordinate transformation

$$\begin{aligned}x &= \rho\sin\phi\cos\theta\\ y &= \rho\sin\phi\sin\theta\\ z &= \rho\cos\phi\\ dV &= (\rho^2\sin\phi)d\rho\ d\phi\ d\theta\end{aligned}$$