Exam 2 - Fall 2018

1. (Midterm 1)

2. (Midterm 1)

3. Maximize $f(x,y) = y$ subject to constant $g(x,y) = (2x-y)^2 + 2(x+3y)^2 = 3$

$\nabla f = \langle 0, 1 \rangle$

$\nabla g = \langle 2(2x-y)2 + 4(x+3y), 2(2x-y)(-1) + 4(x+3y)(3) \rangle$
$= \langle 8x - 4y + 4x + 12y, -4x + 2y + 12x + 9y \rangle$
$= \langle 12x + 8y, 8x + 11y \rangle$

$$\begin{cases} 0 = \lambda(12x + 8y) \\ 1 = \lambda(8x + 11y) \\ (2x-y)^2 + 2(x+3y)^2 = 3 \end{cases}$$

1st Equation $\Longrightarrow$ $(\lambda = 0)$ or $(12x + 8y = 0) \Longrightarrow y = \frac{-12x}{8} = \frac{-3x}{2}$

$\Downarrow$

$1 = 0$ (2nd equation): Contradiction

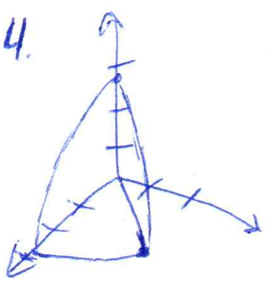
$\Longrightarrow \left(2x + \frac{3x}{2}\right)^2 + 2\left(x + 3\left(\frac{-3x}{2}\right)\right)^2 = 3$

$\Longrightarrow \left(\frac{7x}{2}\right)^2 + 2\left(\frac{-7x}{2}\right)^2 = 3$

$\Longrightarrow 3 \cdot \frac{49x^2}{4} = 3 \Longrightarrow x^2 = \frac{4}{49} \Longrightarrow x = \pm\frac{2}{7}$

Critical points are $(2/7, -3/7)$ and $(-2/7, 3/7)$ $\quad (x_0, y_0) = (-2/7, 3/7)$

4.



$$\int_0^{\sqrt{8}} \int_0^{\pi/4} \int_0^{\pi/2} \frac{2\sin(\rho^2)}{\rho} \rho^2 \sin(\phi)\, d\phi\, d\theta\, d\rho$$

$$= \left(\int_0^{\sqrt{8}} 2\rho \sin(\rho^2)\, d\rho\right)\left(\int_0^{\pi/4} d\theta\right)\left(\int_0^{\pi/2} \sin(\phi)\, d\phi\right) = \left(-\cos(\rho^2)\Big|_0^{\sqrt{8}}\right)\left(\pi/4\right)\left(-\cos(\phi)\Big|_0^{\pi/2}\right)$$

$$= (1 - \cos(8))(\pi/4)(+1) = \frac{+\pi}{4}(1 - \cos(8))$$