MAT 203: CALCULUS III WITH APPLICATIONS

Midterm 2 – April 9, 2018

Spring Semester

Name, Lastname:

ID Number:

Recitation Section: Mon Tue Wed

Directions: The exam starts at 12.00pm and ends at 12:53pm. The exam consists of 4 problems. Calculators and notes are not allowed. Show all relevant work in order to get full credit. In case you need to use the restroom, let the instructor know about this. The back of each page can be used as scratch paper.

Scores	
1	
2	
3	
4	
Total	

Problem 1. Consider the following function of two variables:

$$z = f(x,y) = \cos\left(\frac{1}{x^2+y^2}\right).$$

(1). Make a sketch of the domain of $f(x,y)$.
(2). Find the partial derivatives $f_x(x,y)$ and $f_y(x,y)$.

The values of the partial derivatives at the point $\left(\sqrt{\frac{3}{\pi}}, \sqrt{\frac{3}{\pi}}\right)$ *are*

$$f_x\left(\sqrt{\tfrac{3}{\pi}}, \sqrt{\tfrac{3}{\pi}}\right) = f_y\left(\sqrt{\tfrac{3}{\pi}}, \sqrt{\tfrac{3}{\pi}}\right) = \tfrac{\pi}{36}\sqrt{3\pi}.$$

(3). Find the differential dz at the point $\left(\sqrt{\frac{3}{\pi}}, \sqrt{\frac{3}{\pi}}\right)$ when $dx = 72$ and $dy = 36$.
(4). Find the directions of maximum and minimum increase of $f(x,y)$ at the point $\left(\sqrt{\frac{3}{\pi}}, \sqrt{\frac{3}{\pi}}\right)$. (There is no need to write the directions as unit vectors.)

Problem 2. (1). Write the equation of the sphere S centered at the point $(0,0,1)$ and having radius equal to 4.
(2). Write the equation of the tangent plane to the sphere S at the point $(0,0,5)$.
(3). In a neighborhood of the point $(\sqrt{7},0,4)$ the sphere S can be expressed as the graph of a function of type $z = h(x,y)$. Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $(\sqrt{7},0,4)$.

Problem 3. (1). The length and height of a rectangular box are increasing at rates of 2 in./sec and 1 in./sec., respectively. At what rate is the square of the length of the diagonal of the box increasing when the length is 2 in. and the height is 3 in.?

(2). Use Lagrange multipliers in order to minimize the function $f(x, y) = xy + 8x$ subject to the constraint $x + y = 4$.

Problem 4. (1). Evaluate the integral

$$\int_0^1 \int_0^y x^2 y^{-2}\, dx\, dy.$$

(2). Find the values of each of the following two limits. If a limit does not exist, then write DNE and provide a mathematical justification. If a limit diverges, then write $+\infty$ or $-\infty$ accordingly.

$$a) \quad \lim_{(x,y)\to(0,0)} \frac{\mathrm{e}^{\left(1+2\sqrt{x^2+y^2}\right)} - \mathrm{e}}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{\exp\left(1+2\sqrt{x^2+y^2}\right) - \mathrm{e}}{\sqrt{x^2+y^2}}$$

$$b) \quad \lim_{(x,y)\to(0,0)} \frac{x}{x+y^2}$$