Math 203 - Fall 2018
Final Examination
Thursday, Dec 13, 2018

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This examination contains 17 pages, including this title page, 5 sheets of scratch paper, and a page with relevant identities and formulas. You can tear out the formula sheet and scratch paper if you like.

Read all the questions carefully before starting the exam.

Use of notes, text, calculators, cell phones or computers is not permitted!

Place your final answers in the squares provided!!

Show all your work!!!

Good Luck!!!!

Problem	Score
1	/20
2	/20
3	/20
4	/20
5	/20
6	/20
7	/20
8	/20
9	/20
10	/20
Total	/200

1. Find the unit tangent vector $\mathbf{T}(t)$ and principal normal vector $\mathbf{N}(t)$ for the curve

$$\mathbf{r}(t) = (2\sin(5t), 3t, 2\cos(5t) - 1)$$

at a general point of this curve.

$\boxed{\mathbf{T}(t) =}$

$\boxed{\mathbf{N}(t) =}$

2. Find the length of the curve

$$\mathscr{C} : \mathbf{r}(t) = (4t, t^2 - \ln(t^2)), \quad 1 \leq t \leq 4.$$

$\boxed{\text{Length} = \qquad\qquad\qquad\qquad}$

3. Consider the function

$$F(x, y, z) = e^{xyz} + 3x^2,$$

and the surface

$$\mathcal{S} = \{(x, y, z)\ ;\ F(x, y, z) = 4\}.$$

(a) Find a unit normal vector to $\mathcal{S}$ at the point $(1, 0, 3)$.

Unit normal is

(b) Find the equation for the tangent plane to $\mathcal{S}$ at the point $(1, 0, 3)$.

Tangent plane equation:

4. Find all critical points of the the function

$$F(x,y) = \ln(x^2 + y^2 + 2y + 2)$$

in the domain $D = \{(x,y)\ ;\ x^2 + y^2 < 16\}$, and decide if each is a local minimum, local maximum or saddle point.

Crit.pts (type):

5. Consider the function

$$F(x, y) = x^2 + 2y.$$

Along the set of points (x, y) satisfying the equation

$$x^2 + y^2 = 6y,$$

find the minimum and maximum values of $F(x, y)$ subject to this constraint, and all the points where the minima and maxima are achieved.

Min. value __ is achieved at
Max. value __ is achieved at

6. Compute the integral

$$\iiint_P \frac{z e^{x^2+y^2+z^2}}{x^2+y^2+z^2} dV(x,y,z)$$

over the region

$$P = \{(x,y,z)\ ;\ x^2+y^2+(z-2)^2 \leq 4\}.$$

Answer:

7. Find the surface integral

$$\iint_H f(x,y,z)dS$$

of the function

$$f(x,y,z) = \frac{x^2 z}{(x^2+y^2)^3}$$

over the cylindrical segment

$$H: \quad x^2+y^2=4, \quad \text{and} \quad 0 \le z \le 4.$$

$$\boxed{\text{Integral} = \qquad\qquad\qquad\qquad}$$

8. Compute the integral

$$\int_{\mathscr{C}} 10x\sqrt{1+4y^2}ds$$

over the curve

$$\mathscr{C} : \mathbf{R}(t) = (t^2+5, t), \quad 0 \le t \le 1.$$

$$\boxed{\text{Integral} = \qquad\qquad\qquad\qquad}$$

9. Compute the flux of the vector field

$$\mathbf{F}(x,y) = \langle -x\sin(1+xy) - 4\cos(\pi y), y\sin(1+xy) + 6x\rangle$$

across the curve

$$\mathscr{C}: \ \mathbf{r}(t) = (t, t^2), \quad -1 \le t \le 2$$

in the 'downward' normal direction

$$\mathbf{n}(t) = \frac{1}{\sqrt{4t^2+1}} \langle -2t, -1\rangle .$$

In other words, calculate

$$\int_{\mathscr{C}} \mathbf{F} \cdot \mathbf{n} ds$$

$\boxed{\text{Flux} = \qquad\qquad\qquad\qquad}$

10. Consider the vector field

$$\mathbf{V}(x, y) = \langle y\cos(xy) + \ln(x+1), x\cos(xy) + e^y + x \rangle .$$

Let $\mathscr{C}$ be the circle of radius 2 and center $(0, 4)$, oriented counterclockwise. Compute

$$\oint_{\mathscr{C}} \mathbf{V} \bullet d\mathbf{r}.$$

Answer:

Scratch paper

Scratch paper

Scratch paper

Scratch paper

Scratch paper

Standard Formulas

- Integration-by-parts formula:

$$\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b f'(x)g(x)dx.$$

- Trigonometric summation formula:

$$\begin{aligned}\cos(a+b) &= (\cos a)(\cos b) - (\sin a)(\sin b)\\ \sin(a+b) &= (\cos a)(\sin b) + (\sin a)(\cos b)\end{aligned}$$

- Polar to Cartesian coordinate transformation

$$\begin{aligned}x &= r\cos\theta\\ y &= r\sin\theta\\ dA &= rdrd\theta\end{aligned}$$

- Cylindrical to Cartesian coordinate transformation

$$\begin{aligned}x &= r\cos\theta\\ y &= r\sin\theta\\ z &= z\\ dV &= rdr\ d\theta\ dz\end{aligned}$$

- Spherical to Cartesian coordinate transformation

$$\begin{aligned}x &= \rho\sin\phi\cos\theta\\ y &= \rho\sin\phi\sin\theta\\ z &= \rho\cos\phi\\ dV &= (\rho^2\sin\phi)d\rho\ d\phi\ d\theta\end{aligned}$$