MATH 211, SPRING 2023 PRACTICE MIDTERM 1

FEBRUARY 21

Each problem is worth 10 points.

Problem 1. Find the row reduced echelon form of the matrix A

$$A = \begin{pmatrix} 1 & 0 & 2 & 2 \\ 2 & 1 & 2 & 1 \\ 3 & 0 & 2 & 3 \end{pmatrix}.$$

Problem 2. Decide if the matrix below is invertible, and if so, find its inverse matrix.

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}.$$

Problem 3. Find a basis for the space spanned by the columns of

$$A = \begin{pmatrix} 1 & 3 & 2 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

Problem 4. Find the reflection of $\begin{pmatrix} 2 \\ 3 \\ 0 \\ 4 \end{pmatrix}$ in the hyperplane orthogonal to $\begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}$. Write down the matrix of this transformation, or give the linear map.

Problem 5.

a. Given $T\begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$, $T\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$, and $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$ find the matrix T.

b. Give the matrix of 90 degree counter-clockwise rotation about the positive z axis in $\mathbb{R}^3$.

Problem 6.

a. Find a basis for the kernel and image space of the linear transformation given by

$$A = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}.$$

b. Find $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}^n$ for $n = 1, 2, 3, ...$