MATH 319/320, SPRING 2020 PRACTICE MIDTERM 1

FEBRUARY 27

Each problem is worth 10 points.

Problem 1. Prove by induction

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}.$$

Problem 2. Let (x_n) be an increasing sequence. Prove that (x_n) converges if and only if it is bounded.

Problem 3. Prove that for all positive real numbers $x > 0$ there is an integer n such that $0 < \frac{1}{n} < x$.

Problem 4. State carefully the definition of the supremum of a bounded, non-empty set S of real numbers. Prove that $\sup S = -\inf(-S)$, where $-S = \{-s : s \in S\}$.

Problem 5. Show that there exists a positive real number x such that $x^3 = 2$. Prove that x is irrational.