

Name:

Math 122 (Fall '15)

Midterm 2

November 5, 2015

1. (20pts)	
2. (20pts)	
3. (20pts)	
4. (20pts)	
5. (20pts)	
Total (100pts)	

1. (20pts) Find the derivatives for the following functions:

(1) $v(t) = 3t^2 - 4t + 2$

(2) $y = 17x + 17e^x + 17 \ln x$

(3) $f(u) = e^{u^2+3}$

(4) $w(r) = (r^2 + r^{-2}) \ln r$

(5) $\ln(\ln(\ln x))$

2. (20pts) True/False or Fill-in

- (1) The derivative of the sum of two functions is the sum of their derivatives.
- (2) To compute the derivative of $\ln(\ln x)$, I should use the _____ rule.
- (3) To find the local minima/maxima, I need to compute _____. To find the inflection points, I need to compute _____.
- (4) Using the product rule to differentiate $x^2 \cdot x^2$ gives the same result as differentiating x^4 directly.
- (5) Every critical point of f is either a local maximum or local minimum of f .
- (6) A point p can be both a critical point and an inflection point of a function f .
- (7) If $f'(p) = 0$ and $f''(p) > 0$ then p is a _____ of f .
- (8) The function f has a global maximum on the interval $-5 \leq x \leq 5$ at p if $f(p) \leq f(x)$ for all $-5 \leq x \leq 5$.
- (9) The function f has a global minimum on the interval $-5 \leq x \leq 5$ at p if $f(p) \geq f(x)$ for all $-5 \leq x \leq 5$.
- (10) Every function has a critical point.

3. (20pts) Consider the function $f(x) = x^3 - 3x + 1$ on the interval $-2 \leq x \leq 3$.

(i) Compute $f'(x)$ and $f''(x)$.

(ii) Find all the critical points of f and decide which are local min/max.

(iii) Find the global maximum and minimum of f .

(iv) Find all the inflection points of f .

(v) Graph the function f (on the given interval)

4. (20pts)

I. Find the equation of the tangent line to $f(x) = x \ln x$ at the point $x = e$.

II. During a flood, the water level in a river first rose faster and faster, then rose more and more slowly until it reached its highest point, then went back to its pre-flood level. Consider water depth as a function of time.

(i) Is the time of highest water level a critical point or an inflection point of this function?

(ii) Is the time when the water first began to rise more slowly a critical point or an inflection point?

5. (20pts) I. A yam is put in a hot oven, maintained at a constant temperature 200° C. At time $t = 30$ minutes, the temperature of the yam is 120° C and is increasing at an (instantaneous) rate of 2° C/min. Newton's law of cooling implies that the temperature at time t is given by

$$T(t) = 200 - ae^{-bt}.$$

Find a and b .

II. A grapefruit is tossed straight up with an initial velocity of 50 ft/sec. The grapefruit is 5 feet above the ground when it is released. Its height, in feet, at time t seconds is given by

$$y = -16t^2 + 50t + 5$$

How high does it go before returning to the ground?