Math 122 (Fall '12)

Sample Questions for Midterm 2

1. (20pts) Find the derivatives for the following functions

1. $x^4 + 5x^3 - 2x^2 + 5$

2. $x^{100} + e^{100}$

3. $\sqrt[3]{x} - \frac{1}{\sqrt[3]{x^2}}$

4. $e^t + \ln t$

5. $e^t \cdot \ln t$

6. e^{u^3+u+2}

7. $\sqrt{\ln x + 2}$

8. $s^3 \cdot \ln(e^s + e^{-s})$

9. $\frac{x^2-1}{x^2+1}$

10. x^x (Hint: $x = e^{\ln x}$, and thus $x^x = e^{\ln x \cdot x}$)

2. (10pts) Find the equation of the tangent line to the graph of $y = \ln x$ at $x = e$. Graph the function and the tangent line on the same axes.

3. (10pts) With length, l, in meters, the period T, in seconds, of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{9.8}}.$$

a) How fast does the period increase as l increases? What are units for the rate of change?

b) Does this rate of change increases or decreases as l increases?

4. (10pts) A yam is put in a hot oven, maintained at a constant temperature 200° C. At time $t = 30$ minutes, the temperature of the yam is 120° C and is increasing at an (instantaneous) rate of 2° C/min. Newton's law of cooling implies that the temperature at time t is given by

$$T(t) = 200 - ae^{-bt}.$$

Find a and b.

5. (20pts) Graph the function

$$f(x) = x^3 - 3x^2 + 2$$

Your answer should include:

a) Local maxima/minima,

b) Inflection points.

6. (20pts) The derivative of $f(t)$ is given by $f'(t) = t^3 - 6t^2 + 8t$ for $0 \leq t \leq 5$.

i) Graph $f'(t)$, and describe how the function $f(t)$ changes over the interval $t \in [0, 5]$.

ii) When is $f(t)$ increasing and when is it decreasing?

iii) Where does $f(t)$ have a local maximum and where does it have a local minimum?

iv) What are the inflection points of f?

7. (10pts) When I got up in the morning I put on only a light jacket because, although the temperature was dropping, it seemed that the temperature would not go much lower. But I was wrong. Around noon a northerly wind blew up and the temperature began to drop faster and faster. The worst was around 6pm when, fortunately, the temperature started going back up.

a) When was there a critical point in the graph of temperature as a function of time?

b) When was there an inflection point in the graph of temperature as a function of time.