Spring 2017 MAT 536, Complex Analysis

Instructor: Samuel Grushevsky

Homework #10, due in class Wed April 19

Recall that $\Omega \subsetneq \mathbb{C}$ is open connected, $\mathcal{O}(\Omega)$ denotes the set of holomorphic functions on Ω, $\mathcal{H}(\Omega)$ denotes the set of harmonic functions on Ω, and $\mathcal{SH}(\Omega)$ denotes the set of subharmonic functions on Ω; $\Delta_r(z)$ denotes the open disk around z of radius r.

Problem 1 (double credit)**.** Suppose $u \in \mathcal{H}\Big(\Delta_R(0)\setminus\{0\}\Big)$ is such that $\lim_{z\to 0} zu(z) = 0$. Prove that there exists a function $u_0 \in \mathcal{H}(\Delta_R(0))$, and a constant $\alpha \in \mathbb{R}$ such that $u(z) = \alpha \log|z| + u_0(z)$ for any $z \in \Delta_R(0)\setminus\{0\}$. (*See the hints in Ahlfors, where this is exercise 8* in paragraph 6.4 of chapter 4*)

Problem 2. Prove that if a sequence of positive harmonic functions on Ω converges pointwise, then it converges uniformly on compact subsets of Ω.

Problem 3. For any $f \in \mathcal{O}(\Omega)$ and for any $\alpha \in \mathbb{R}_{\geq 0}$, prove that $|f|^\alpha \in \mathcal{SH}(\Omega)$.

Problem 4. Suppose $u \in \mathcal{SH}(\Omega)$, while $F : \mathbb{R} \to \mathbb{R}$ is a convex function. Prove that $F \circ u \in \mathcal{SH}(\Omega)$.