Spring 2017 MAT 536, Complex Analysis Instructor: Samuel Grushevsky Homework #6, due in class Wed March 22

Problem 1. How many roots does the equation $3z^5+21z^4+5z^3+6z+7$ have in the unit disk? (*) Does it have a multiple root there?

Problem 2. Compute the integral

$$\int_0^\infty \frac{x^2 dx}{x^4 + x^2 + 1}.$$

Problem 3. Compute the integral

$$\int_0^\infty \frac{\cos x - 1}{x^2} dx.$$

Problem 4. Compute the integral

$$\int_0^\infty \frac{x^{1/3}}{1+x^2} dx.$$

Problem 5. Use the residue theorem to evaluate the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$