Spring 2017 MAT 536, Complex Analysis

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Homework #7, due in class Wed March 29

Problem 1. Find all possible values of $\int_\gamma \frac{dz}{1+z^2}$ for γ being any smooth closed curve in $\mathbb{C}$ not passing through $\pm i$.

Problem 2. Suppose $\Omega \subset \mathbb{C} \setminus \{0\}$ is any open simply connected set. Show that there exists a single-valued branch of the function z^a on Ω, for any $a \in \mathbb{R}$.

Problem 3. Let $f : \Delta \to \Delta$ be a holomorphic map, and let d be the hyperbolic metric on Δ. Use the Schwarz lemma to prove that for any $z_1, z_2 \in \Delta$, the inequality $d(f(z_1), f(z_2)) \le d(z_1, z_2)$ holds.

(*This is not just to remind you of the midterm, and to make you look up the formula for the hyperbolic metric. You may be curious to learn more about the Kobayashi (pseudo)metric*)

Problem 4. Show that the function $1/z^2$ cannot be uniformly approximated by polynomials on the annulus $A = \{1/2 \le |z| \le 2\}$.

Problem 5. Suppose $f \in \mathcal{O}(\Delta)$, and suppose the series $f(z) + f'(z) + f''(z) + \ldots$ converges at $z = 0$. Prove that f can be extended to a function in $\mathcal{O}(\mathbb{C})$.