Spring 2017 MAT 536, Complex Analysis
Instructor: Samuel Grushevsky
Homework #8, due in class Wed April 5

Problem 1. Give an example of an equicontinuous, but not normal, family of functions $f : \Delta \to \mathbb{R}$.

Problem 2. Give an example of a normal, but not sequentially closed, family of functions, contained in $\mathcal{O}(\Omega)$, for some domain Ω.

Problem 3 (Hadamard's three circle theorem)**.** Suppose

$$f \in \mathcal{O}(\Delta_R(0) \setminus \Delta_r(0))$$

is a holomorphic function on an annulus. Denote

$$M(\rho) := \max_{|z|=\rho} |f(z)|$$

for any $r < \rho < R$, and denote $a := \log \rho$. Prove that $\log M(e^a)$ is a convex function of a.

Problem 4. For any continuous function $f : \partial\Delta \to \mathbb{R}$ on the unit circle, prove that for any $z \in \partial\Delta$ the limit

$$\lim_{a \to z} \int_{z \in \partial\Delta} P_a(z) f(z) \frac{dz}{iz},$$

where $P_a(z)$ is the Poisson kernel, is equal to $f(z)$.