MAT319

Midterm II. November 4, 2014

This is a closed notes/ closed book/ electronics off exam.

You are allowed and encouraged to motivate your reasoning, but at the end your proofs should be formal logical derivations, whether proving that something holds for all, or proving that your example works.

You can use any theorem or statement proven in the book; please refer to it in an identifiable way, eg. "by the limit law for the sum of two sequences", "by the bounded monotonic convergence theorem", etc.

Please write legibly and cross out anything that you do not want me to read.

There are **five** *problems on this midterm, each worth 25 points (though the problems are of variable difficulty, I think #5 is the hardest), and your grade will be the sum of your scores for the* **four best** *problems*

Name:						
Problem	*1*	*2*	*3*	*4*	*5*	*Total*
Grade						

Problem 1. Let (s_n) be the sequence of partial sums of a series $\sum a_n$. If the series diverges, can (s_n) be

a) a bounded sequence?

b) a bounded increasing sequence?

[if yes, provide an example, if not, prove why not]

Problem 2. a) State the *Intermediate Value Theorem* for continuous functions $f : \mathbb{R} \to \mathbb{R}$.

b) Prove that the polynomial $x^5 + 10x^4 + 1$ has a root.

Problem 3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ |x| & \text{if } x \notin \mathbb{Q} \end{cases}$$

For which $x_0 \in \mathbb{R}$ does the limit $\lim_{x \to x_0} f(x)$ exist?

Problem 4. Suppose a function $f : \mathbb{R} \to \mathbb{R}$ satisfies the following property:

$$\forall \varepsilon > 0,\ \exists \delta > 0,\ \forall x, y \in \mathbb{R},\ \ |x-y| < \delta \Rightarrow |f(x)-f(y)| < \varepsilon \qquad (*)$$

(in words: for any positive number ε there exists a positive number δ such that for any pair of real numbers x, y the inequality $|x - y| < \delta$ implies $|f(x) - f(y)| < \varepsilon$)

Does it then follow that f is continuous on $\mathbb{R}$?

Problem 5. Construct a *continuous* function $f : \mathbb{R} \to \mathbb{R}$ that does *not* satisfy property $(*)$ from the previous problem.