## MAT 118 Spring 2017

## Practice Exam for Midterm \#1

To be reviewed in class on Monday, 2/27/17

1. Consider this preference schedule for an election between candidates $\mathrm{A}, \mathrm{B}$ and C :

| \# voters: | 5 | 4 | 3 |
| :---: | :---: | :---: | :---: |
| 1st | A | C | B |
| 2nd | B | B | C |
| 3rd | C | A | A |

(a) Which candidate wins using the plurality method?
(b) Use the weighted Borda count method to determine the outcome, in which 1st place votes are each 4 points, 2 nd place votes are 2 points, and 3rd place votes are 1 point.
(c) Use the plurality-with-elimination method to determine the outcome.
(d) Finally, apply the method of pairwise comparisons to determine the outcome.
(e) Is there a Condorcet candidate? If so, who is it?
2. Consider the weighted voting system $[20 ; 7,6,4,3,1]$ with players $P_{1}, P_{2}, P_{3}, P_{4}, P_{5}$.
(a) Does any player have veto power? If so, how many and which ones?
(b) Compute the Banzhaf power distribution.
(c) How many sequential coalitions are there in this weighted voting system?
3. Compute the Shapley-Shubik power distribution of $[5 ; 4,2,1]$.
4. Andy and Brianna are dividing a giant rectangular ice cream cake using the divider-chooser method. After flipping a coin, they decide that Andy will be the divider. The cake has 3 flavors: lemon, strawberry and key lime. Here's an illustration of the cake:


Andy likes does not like strawberry at all and likes lemon twice as much as key lime, while Brianna likes all flavors equally.
(a) Make a table listing the values, relative to the total value of the cake, of each flavor according to each of Andy and Brianna. Use either fractions or percentages.
(b) Describe, with pictures and in words (i.e. fractions of flavors), two distinct ways that Andy can divide the cake, with the following restriction: at least one of the divisions must allow Brianna to leave with more than a minimal fair share, in her opinion.
5. In problem \#5, suppose that we have a third player, Casey, in addition to Andy and Brianna. Casey only likes lemon. Now suppose the 3 players decide to use the Lone-divider method.
(a) Write a table, enlarging the table from 6(a), listing the values, relative to the total value of the cake, of each flavor according to the 3 players. Use either fractions or percentages.
(b) Suppose Brianna is the divider, and that she divides the cake along the flavor lines, so that one piece is all lemon, one is all strawberry, and one is all key lime. What are the bidding lists (i.e. fair share lists) of Andy and Casey?
(c) Describe how this instance of the Lone-divider method might be completed.
6. Describe how the Lone-Chooser method might proceed with the cake in \#6, in which Casey is the chooser. In your description, how much value does she leave with?
7. Aleksander and Brittany are getting divorced. They are going to use the method of sealed bids to decide who gets their vacation property on Long Island. The bids, when revealed, are:

| Aleksander | Brittany |
| :---: | :---: |
| $\$ 450,000$ | $\$ 300,000$ |

Describe how the method of sealed bids proceeds. Be sure to record the fair share values of each player, the first settlement, the surplus, and the final settlement.
8. Three friends (A, B and C) are dividing candy that they collected Halloween night. They use the method of markers, and they place the markers as follows:


Carry out the method of markers to determine who gets what. How many pieces of candy does each player end up with?

