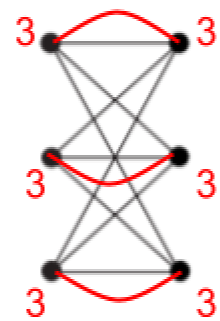
MAT 118 Spring 2017

Practice Exam for Midterm #2

With solutions!

1. Consider the following graph:



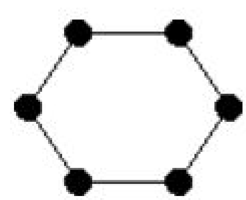
   (a) (Write the degrees of the vertices on the graph. See picture above.

   (b) Does this graph have any Euler circuits? Why or why not? No: the graph does not have all of its vertices even, so by Euler's Circuit Theorem, it has no Euler Circuits.

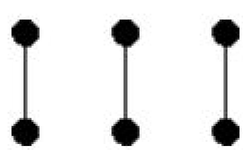
   (c) Does this graph have any Euler paths? Why or why not? No: the graph has more than two odd vertices, so by Euler's Path Theorem, it has no Euler Paths.

   (d) Give an optimal Eulerization of the graph. How many edges did you add? See picture above.

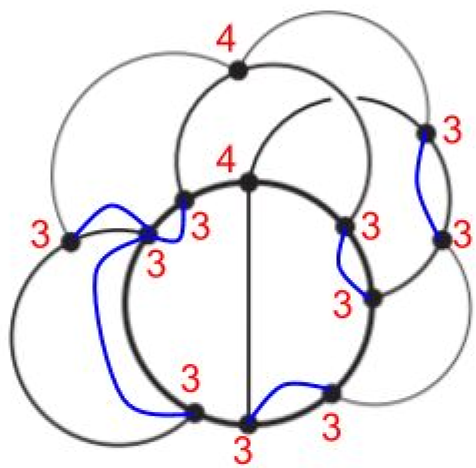
2. (a) Give an example of a connected graph with 6 vertices, each vertex of degree 2. Answer:



   (b) Give an example of a graph with 6 vertices, each vertex of degree 1. Answer:



3. Consider the following graph:



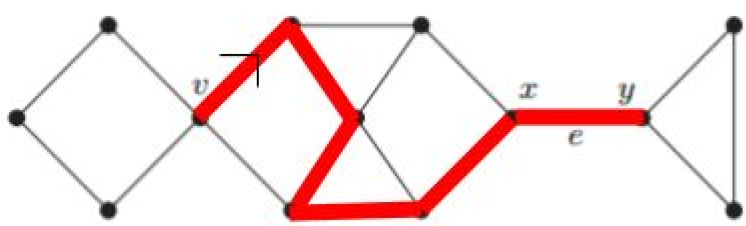
(a) Write the degrees of the vertices on the graph. See picture above.

(b) Does this graph have any Euler circuits? Why or why not? No: same answer as 1(b).

(c) Does this graph have any Euler paths? Why or why not? No: same answer as 1(c).

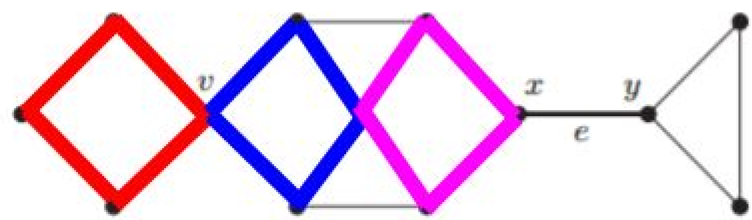
(d) Give an Eulerization of the graph. See picture above (in blue). There are other solutions.

4. Consider the following graph:

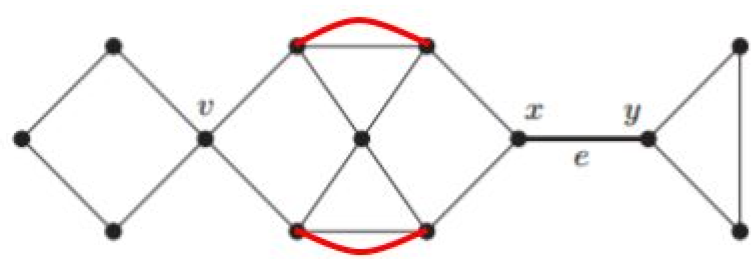


(a) Find a path from v to y of length 6. See picture above.

(b) Find all circuits in the graph of length 4. How many are there? There are 6 total; only 3 if we ignore directions. Below are the 3 directionless circuits, each in a different color.



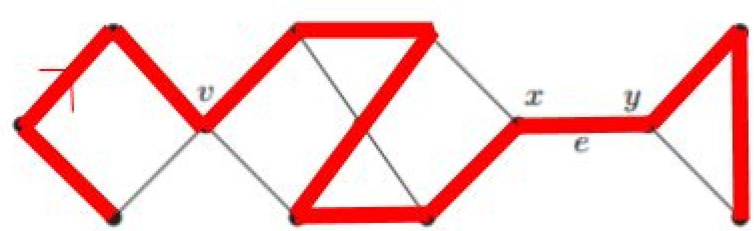
(c) Semi-eulerize the graph, leaving x and y as the two distinguished odd vertices. Here:



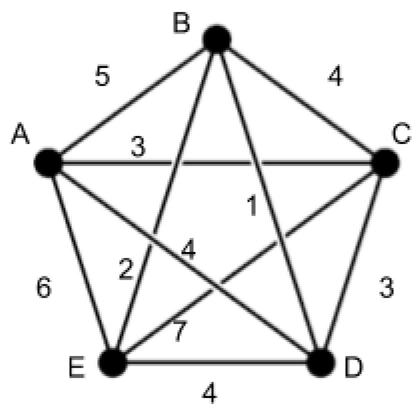
5. For each of the following, if your answer is "yes" then draw an example, and if your answer is "no" then give a brief explanation.

(a) Does the graph in problem #4 have any Hamilton circuits? No; impossible to hit every vertex without retracing some steps.

(b) Does the graph in problem #4 have any Hamilton paths? Yes; here's an example:



6. Consider the following weighted graph:

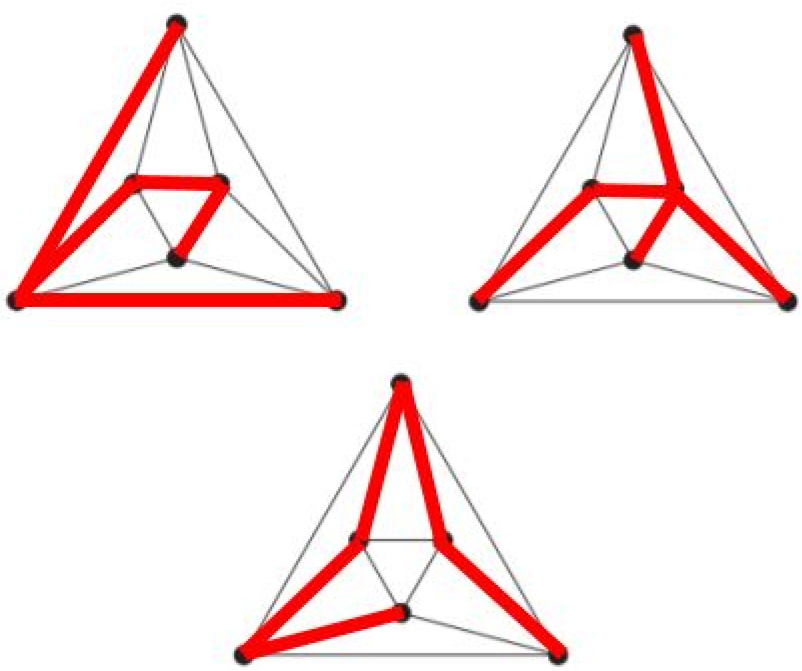


(a) Using the NNA starting at A, give the resulting Hamilton circuit and its total weight. The Hamilton circuit we get is A, C, D, B, E, A. The total weight is 3 + 3 + 1 + 2 + 6 = 15.

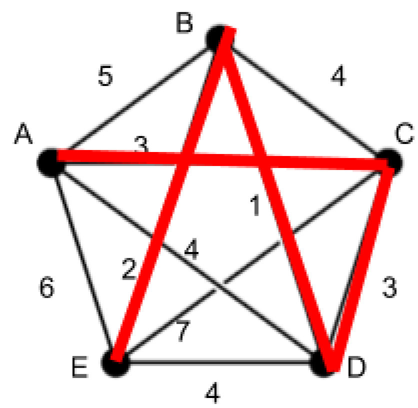
(b) Using the NNA starting at B, give the resulting Hamilton circuit and its total weight. The Hamilton circuit is B, D, C, A, E, B. This is the reversal of (a): the total weight is again 15.

(c) Using the Cheapest link algorithm, give the resulting Hamilton circuit and its total weight. The Hamilton circuit is that of (a) or (b), up to direction; the total weight is also 15.

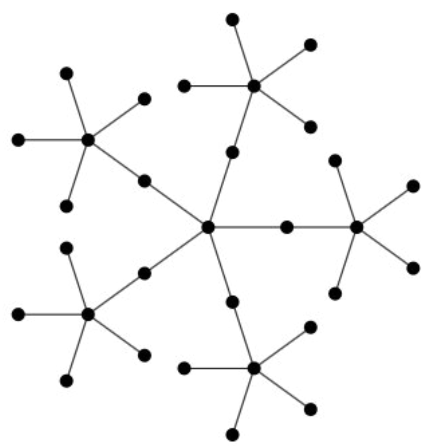
7. For the following graph, produce 3 different spanning trees. Here are 3 (there are many more):



8. For the weighted graph in problem #6, reproduced below, use Kruskal's algorithm to find an MST, and give its total weight. The output is indicated in the picture. The total weight is 1 + 2 + 3 + 3 = 9.



9. (a) How many spanning trees does the following graph have? Just 1; the graph is a tree.



(b) If each edge has weight 3, what's the total weight of an MST? An MST must be the whole graph, by part (a). The graph has 30 edges, so the total weight is 3 x 30 = 90.