

MAT 118 1/25/17

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Ch 1: 1.3, 1.4 Borda Count Method,
Plurality with elimination method.

Last time we discussed the plurality method.

Today we discuss two other voting methods.

Borda Count Method

This method aims to use all the information on the preference ballots to give a final ranking of the candidates.

Typically, in an election with N candidates, the Borda Count method assigns

N points to each 1st place vote
 $N-1$ points to each 2nd place vote
⋮
1 point to each last place vote.

② "The outcome" of the election is a ranking of the candidates ordered by the #points that they each have.

Recall from lecture 1:

#votes	2	8	6	11	3
1	A	A	B	C	D
2	B	D	D	D	B
3	C	C	A	B	A
4	D	B	C	A	C

We calculate the #points for each candidate:

$$A: 10 \times 4 + 9 \times 2 + 11 \times 1 = 40 + 18 + 11 = 69$$

$$B: 6 \times 4 + 5 \times 3 + 11 \times 2 + 8 \times 1 = 24 + 15 + 22 + 8 = 69$$

$$C: 11 \times 4 + 10 \times 2 + 9 \times 1 = 44 + 20 + 9 = 73$$

$$D: 3 \times 4 + 25 \times 3 + 2 \times 1 = 12 + 75 + 2 = 89$$

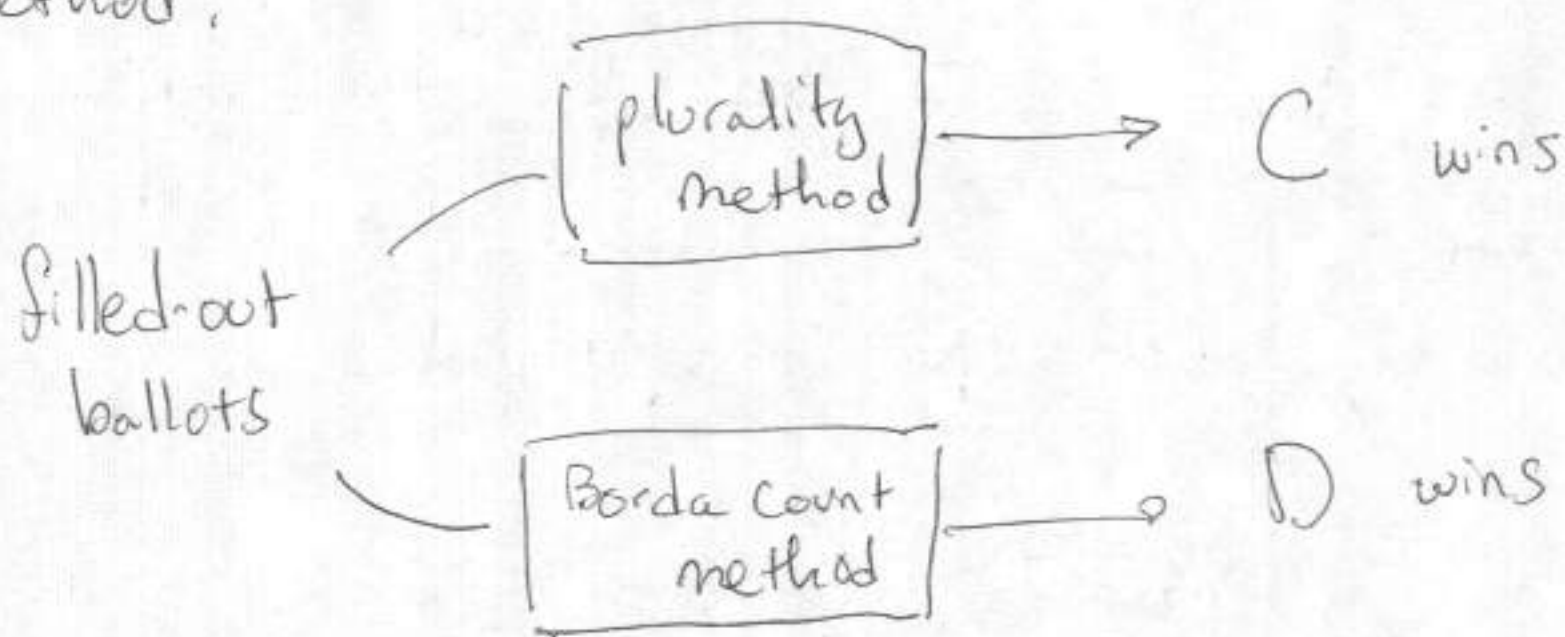
The final ranking is thus:

1st — D (89 pts)

2nd — C (73 pts)

3rd/4th (tie) — A & B. (69 pts each)

So we have two different outcomes, depending on the method!



modified Borda Count:

Sometimes we want to distribute points differently. In general, we can choose

N numbers $n_1, n_2, n_3, \dots, n_N$

④ Such that $n_1 > n_2 > n_3 > \dots > n_N$

and

1st place vote = n_1 points

2nd place vote = n_2 points

etc.

In the standard Borda count method,

we take $n_1 = N, n_2 = N-1, \dots, n_N = 1$.

In the text there is an example, the voting method for the Cy Young award for pitchers in baseball, with $N=5$ candidates and

$n_1 = 7, n_2 = 4, n_3 = 3, n_4 = 2, n_5 = 1$.

This choice of point values emphasizes the value of 1st place over any other kind of vote.

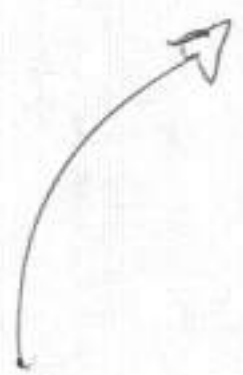
The Plurality with Elimination Method

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One drawback of the plurality method is that a candidate can win without a majority ($> 50\%$) of votes.

The first way one may solve this is to use a voting method implemented in many TV shows such as X-factor, American Idol:

- have a single choice ballot election and eliminate the candidate with fewest votes
- then repeat, with remaining candidates
- eventually, 2 candidates are left, and in the final election the winner has a majority of votes.
(if there is no tie.)

 This is the Runoff method.

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The runoff method can be costly and/or time consuming, so instead the following modification is implemented:

plurality with elimination (or "instant runoff method")

For this the voters vote only once, but using a preference ballot.

- Steps
- if a candidate has $> 50\%$ of 1st place votes, they win.
 - Otherwise, eliminate the candidate with the fewest 1st place votes.
 - transfer the votes of the eliminated candidate to the next eligible candidate
 - repeat, until 1 candidate has $> 50\%$

Example, using our previous preference schedule:

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round 1:

A	B	C	D
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10	6	11	3
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← # 1st place votes

No majority. D has the fewest votes (3), and these are transferred to B.

round 2:

A	B	C	D
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10	9	11	
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No majority, and B has the fewest votes (9), and these are transferred to A.

round 3:

A	B	C	D
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19		11	
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A has a majority, so wins!

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In our example, 3 different voting methods have led to 3 different outcomes.

