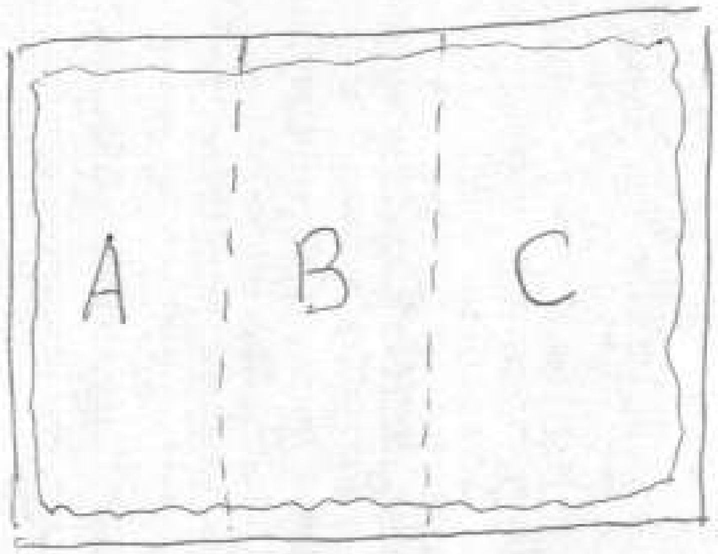
One more lone divider example: (1)

This time let's have a divider D and two choosers C_1 and C_2 try to fairly divide a Sicilian (square-shaped) pizza:



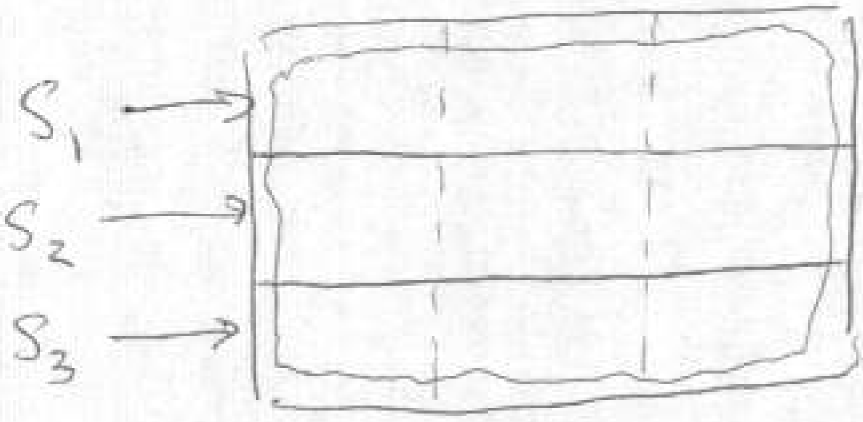
A = anchovies   B = basil   C = chicken

Here are the value systems of the 3 players:

	A	B	C
D	1/3	1/3	1/3
C_1	1/2	1/2	0
C_2	0	1/2	1/2

The first step is for the divider D to divide the pizza into what he/she consides 3 equal parts.

(2) Here is one valid way to do this:



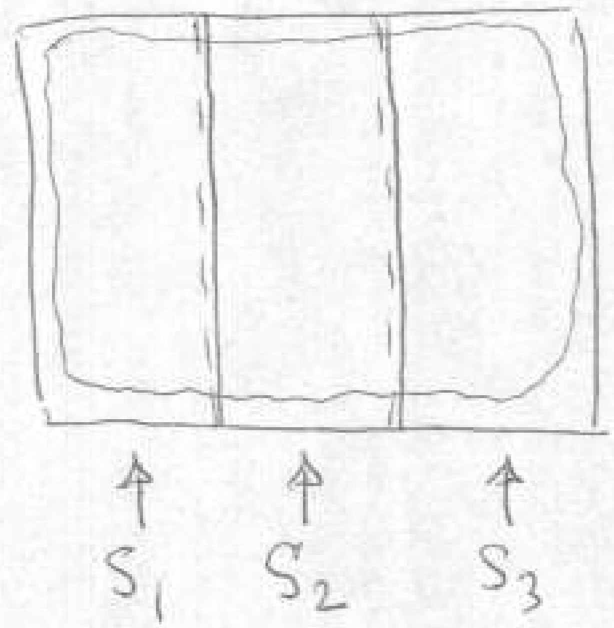
and here is the value table for this division:

	S_1	S_2	S_3
D	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
C_1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
C_2	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Question: how many ways are there to fairly distribute S_1, S_2, S_3?

Well we can do it any way we want. You can write all the ways down or you might realize that it is just the number of ways of ordering S_1, S_2, S_3 (whichever one is first goes to D, the next goes to C_1, and the 3rd to C_2). Thus the number of ways is equal to $3! = 3 \times 2 \times 1 = 6$.

Here is another valid move for D:



So that S_1 is all anchovies, S_2 all basil, and S_3 all chicken. The new value table is:

	S_1	S_2	S_3
D	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
C_1	$\frac{1}{2}$	$\frac{1}{2}$	0
C_2	0	$\frac{1}{2}$	$\frac{1}{2}$

which is just the original value systems table. Now not every share is fair to everyone.

	fair shares
D	$\{S_1, S_2, S_3\}$
C_1	$\{S_1, S_2\}$
C_2	$\{S_2, S_3\}$

(4)

Question: How many ways are there to fairly distribute S_1, S_2, S_3?

We list all the ways.

D	S_1	S_2	S_3
C_1	S_2	S_1	S_1
C_2	S_3	S_3	S_2

(this is one fair division, in which D gets S_1, C_1 gets S_2, C_2 gets S_3)

Thus there are 3 ways.

Finally, let's look at a case where there is a "standoff." Let's suppose instead that C_1 and C_2 both only like basil. Here's the new value table:

	A	B	C
D	$1/3$	$1/3$	$1/3$
C_1	0	1	0
C_2	0	1	0

Suppose D divides just as before, so that

S_1 = all anchovies

S_2 = all basil

S_3 = all chicken.

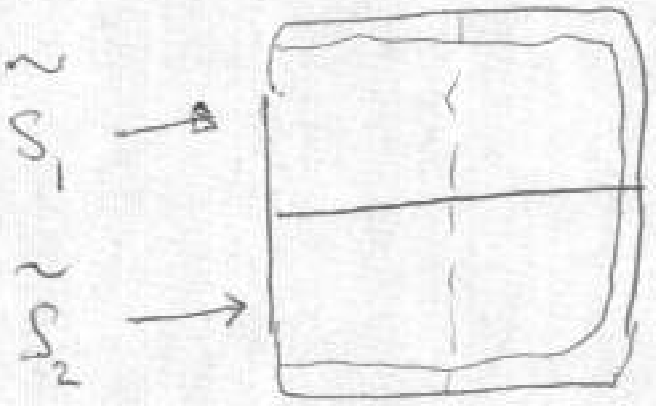
Then the value table for S_1, S_2, S_3 is the same table we just wrote down.

In particular, there is no fair distribution — both C_1 and C_2 only want S_2 = basil.

So the next step in the lone-divider method tells us that we give D a piece other than S_2, say S_1, and recombine S_2 and S_3:

Now we use the divider-chooser method with this and using only the 2 players C_1 and C_2.

We suppose after a coin toss that C_1 is the new divider, and makes the cut:



Now $\tilde{S}_1$ and $\tilde{S}_2$ are both worth $\frac{1}{2}$ to C_1, C_2

So they are each happy with either.

C_1 can get $\tilde{S}_1$ and C_2 can get $\tilde{S}_2$, for example.

Question: How much value did each player walk away with?

D got S_1, worth $\frac{1}{3}$ (to him/her)

C_1 got $\tilde{S}_1$, worth $\frac{1}{2}$!

C_2 got $\tilde{S}_2$, worth $\frac{1}{2}$ also!

D got the minimal guaranteed fair value but C_1 and C_2 walked away with substantially more value.

The lone-chooser method (3 players)

Step 0: Among the 3 players, choose one "chooser" C and two "dividers" D_1 and D_2.

Step 1: D_1 and D_2 use the divider-chooser method to divide the asset(s).

Step 2: Each of D_1 and D_2 divides his/her share into 3 equal (in his/her opinion) subshares.

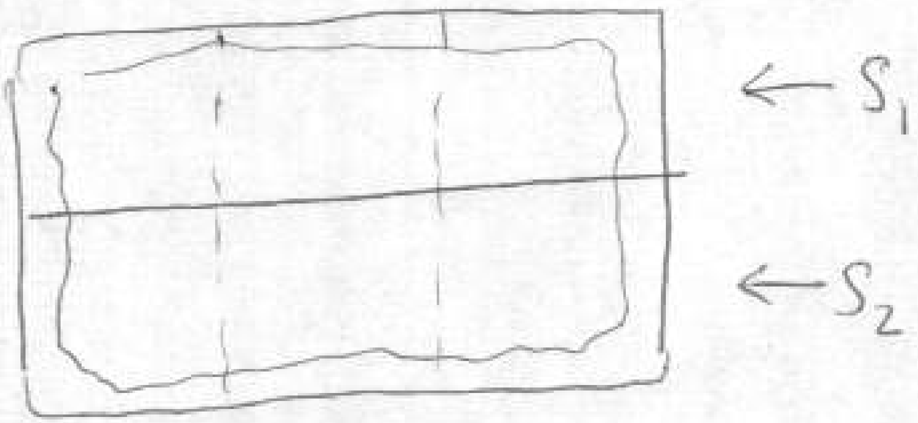
Step 3: Then C chooses one subshare from D_1 and one from D_2, and that's it.

Example: let's return to our pizza.

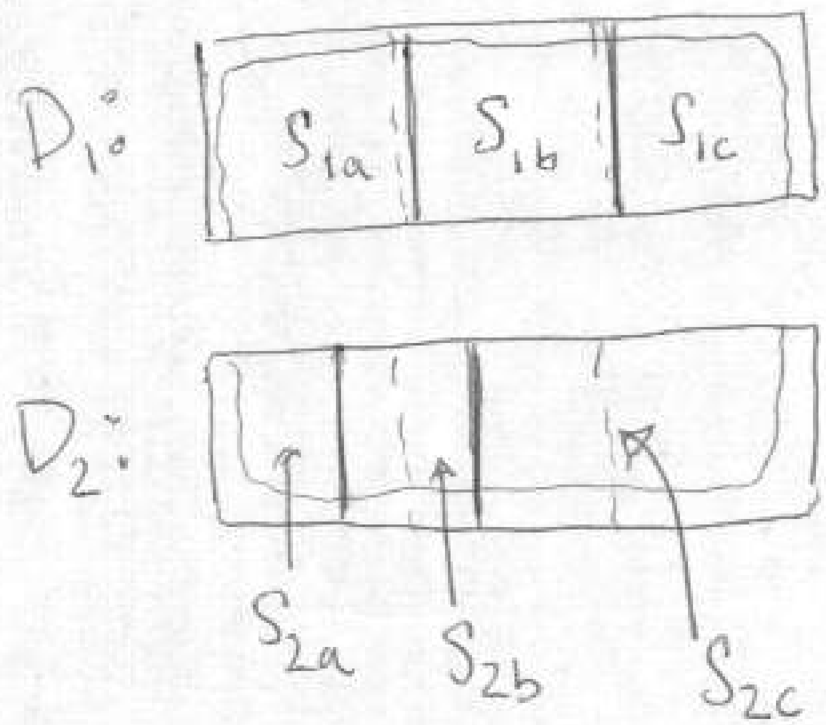
and suppose these are the value systems of the 3 players:

	A	B	C
D_1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
D_2	$\frac{1}{2}$	$\frac{1}{2}$	0
C	0	$\frac{1}{2}$	$\frac{1}{2}$

Suppose when D_1 and D_2 apply divider-chooser, that D_1 is the divider, and the cut is:



Both top and bottom are equal to D_2, so D_2 is fine taking either. Let's say D_1 takes the top, D_2 the bottom. Then each of them has to divide their share into 3 equal parts (in their opinions). Here is a valid way:



Now C gets to choose one piece from D_1 and one piece from D_2.