The method of sealed bids

We start with an example. Suppose there are 3 siblings inheriting an estate left to them by their grandfather: Allen, Barbara & Chip.

There are 3 items included in the estate: a car, a boat, and a house.

In order to divide the estate, the acting lawyer suggests using the method of sealed bids.

Each sibling writes down, in secret, what they are willing to pay (or what they consider each item to be worth in dollars). They then seal their "bids" in envelopes. The lawyer then opens all the envelopes and records the bids:

	Allen	Barbara	Chip
Car	\$60,000	\$75,000	\$30,000
Boat	\$15,000	\$45,000	\$60,000
House	\$825,000	\$330,000	\$810,000

The first step of this method is for the lawyer to allocate the items to the highest bidders: for each item, whoever bid most receives the item

The car goes to Barbara.

The boat goes to Chip.

The house goes to Allen.

Next, the lawyer computes the "First settlement", as follows:

We define a person's <u>fair-share</u> value of the estate to be the total value of all the bids of that person, divided by the number of people (in this case, 3). For example:

$$\text{Allen's fair share value} = \left(\$60{,}000 + \$15{,}000 + \$825{,}000\right) / 3$$

$$= \$900{,}000 / 3$$

$$= \$300{,}000.$$

Similarly, we compute:

$$\text{Barbara's fair share value} = (\$75{,}000 + \$45{,}000 + \$330{,}000)/3$$
$$= \$450{,}000/3$$
$$= \$150{,}000$$

$$\text{Chip's fair share value} = (\$30{,}000 + \$60{,}000 + \$810{,}000)/3$$
$$= \$900{,}000/3$$
$$= \$300{,}000$$

These numbers represent what each person considers a fair amount for them to walk away with.

Now using these numbers the lawyer computes how much money each person "owes" (temporarily) to the estate (or, if the result is negative, how much the estate owes the person).

(4) The equation is:

$$\begin{pmatrix}\text{value of}\\ \text{items received}\end{pmatrix} - \begin{pmatrix}\text{fair share}\\ \text{value}\end{pmatrix} = \begin{bmatrix}\text{amount owed}\\ \text{to the estate}\\ \text{if positive}\\ \hline \text{amount the}\\ \text{estate owes}\\ \text{if negative}\end{bmatrix}$$

For Allen we compute:

$$\underset{\text{(house)}}{\$825{,}000} - \underset{\text{(fair share)}}{\$300{,}000} = \$525{,}000$$

So (since the number is positive) Allen owes \$525,000 to the estate.

For Barbara:

$$\underset{\text{(car)}}{\$75{,}000} - \underset{\text{(fair share)}}{\$150{,}000} = -\$75{,}000$$

So (since the result is negative) Barbara is owed \$75,000 by the estate.

For Chip:

$$\underset{\text{(boat)}}{\$60{,}000} - \underset{\text{(fair share)}}{\$300{,}000} = -\$240{,}000$$

So Chip is owed \$240,000 by the estate.

Here is the first settlement:

Allen: House, gives away \$525,000

Barbara: Car, gets \$75,000

Chip: Boat, gets \$240,000

We use Allen's money to pay Barbara & Chip. But there's some left over! The <u>surplus</u> is

$$\$525{,}000 - \$75{,}000 - \$240{,}000$$

$$= \$210{,}000.$$

The <u>final settlement</u> is obtained by dividing this surplus equally among the 3 siblings.

(6)

$$\text{Surplus}/3 = \$210{,}000/3 = \$70{,}000.$$

Now instead giving away \$525,000, Allen only gives away \$525,000 - \$70,000 = \$455,000

And Barbara now gets \$75,000 + \$70,000

= \$145,000,

while Chip gets \$240,000 + \$70,000

= \$310,000.

The end result:

Allen: gets the House and pays \$455,000

Barbara: gets the car and receives \$145,000

Chip: gets the boat and receives \$310,000