Another example from the method of sealed bids: (Example 3.11 of the text)

Al and Betty are getting divorced. The only joint property of any value is their house. They give the method of sealed bids a try. The bids:

Al: \$340,000 Betty: \$364,000

So Betty gets the house. Let's compute the fair share values for Al and Betty:

$$\text{fair share value of Al} = \frac{\$340{,}000}{2} = \$170{,}000$$

$$\text{fair share value of Betty} = \frac{\$364{,}000}{2} = \$182{,}000$$

Thus Al is owed \$170,000 by the estate, and Betty owes \$364,000 − \$182,000 = \$182,000.

(2) There is a surplus of

$$\$182{,}000 - \$170{,}000 = \$12{,}000.$$

Dividing this by 2, we give each of Al and Betty \$6,000. So the final settlement is:

Al: gets \$176,000

Betty: gets the house, pays \$176,000.

An example showing the pitfall of overbidding: 3 people are trying to divide the assets of a nice car and a huge, new TV. Call the players P_1, P_2 and P_3. They are each instructed to write down their bids and seal them. P_1, not understanding the method of sealed bids very well, knows at least that if his or her bids are highest, then the items go to him/her. Although P_1 really values the car at about \$20,000 and the TV at \$1,000, he/she decides to bid higher to guarantee that he/she wins the items. The bids are unsealed and we have:

	P_1	P_2	P_3
Car	\$80,000	\$23,500	\$20,000
TV	\$10,000	\$500	\$1,000

(So P_3 actually bid the way P_1 would have, if P_1 was being honest!)

So P_1 succeeds in getting the car and TV.
But let's see what happens:

Compute the fair share values:	P_1	P_2	P_3
	\$30,000	\$8,000	\$7,000

Note that P_1's "True" fair share value is equal to \$7,000, which is P_3's fair share value.

So P_1 owes \$80,000 - \$30,000 = \$50,000

(4) while P_2 is owed \$8,000,

and P_3 is owed \$7,000.

The surplus is \$50,000 - \$8,000 - \$7,000 = \$35,000,

and $\frac{\$35{,}000}{3} = \$11{,}667$.

The final settlement:

P_1 gets the car and TV

and pays \$50,000 - \$11,667 = \$38,333

P_2 gets \$8,000 + \$11,667 = \$19,667

P_3 gets \$7,000 + \$11,667 = \$18,667

P_2 and P_3 got more than double their fair share values! According to how P_1 bid, P_1 got

$$\underbrace{\$80{,}000}_{\substack{\text{CAR}\\ \text{(fake)}}} + \underbrace{\$10{,}000}_{\substack{\text{TV}\\ \text{(fake)}}} - \underbrace{\$38{,}333}_{\text{owed cash}} = \$51{,}667$$

in value.

However, P_1 honestly believed the car was \$20,000 and the TV was \$1,000, so truthfully P_1 got

$$\underset{\substack{\text{CAR}\\(\text{true})}}{\$20{,}000} + \underset{\substack{\text{TV}\\(\text{true})}}{\$1{,}000} - \underset{\text{owed each}}{\$38{,}333} = -\$17{,}333$$

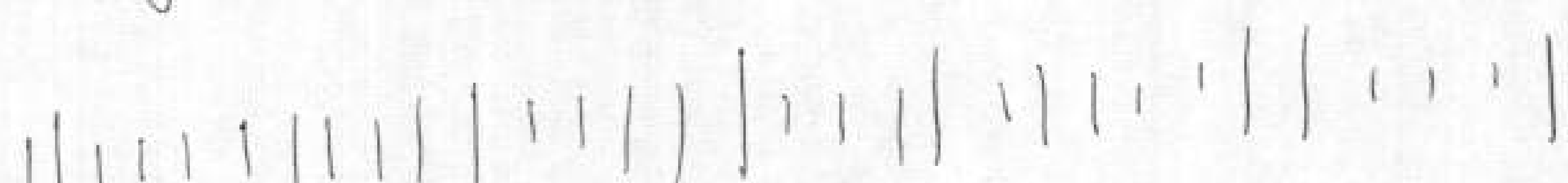
So really P_1 lost a bunch of money!

The method of markers

This is the last division method we will discuss, and it works for discrete assets with a large number of different items. The prototypical example is that of dividing Halloween candy.

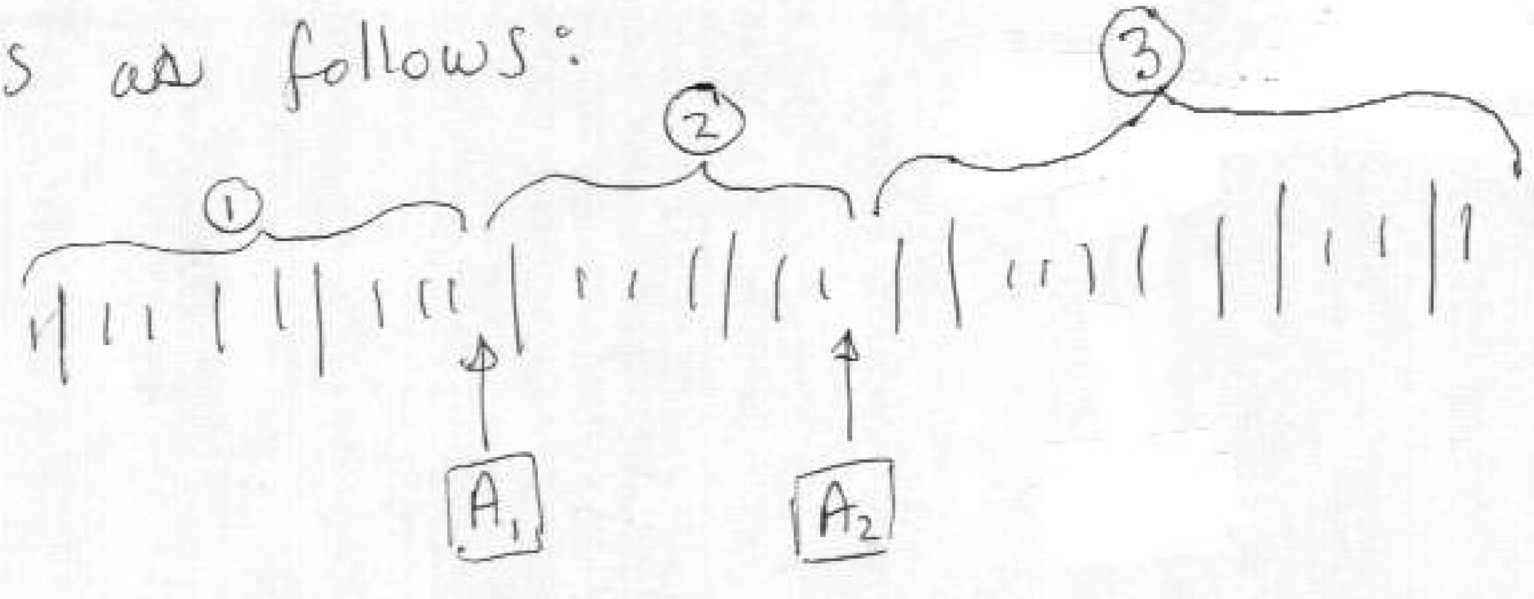
We suppose that there are 3 people dividing the candy, and that the candy is arranged in a single line from left to right, which we call an <u>array</u>.

The array of candy:



(imagine each line is a kind of candy bar).

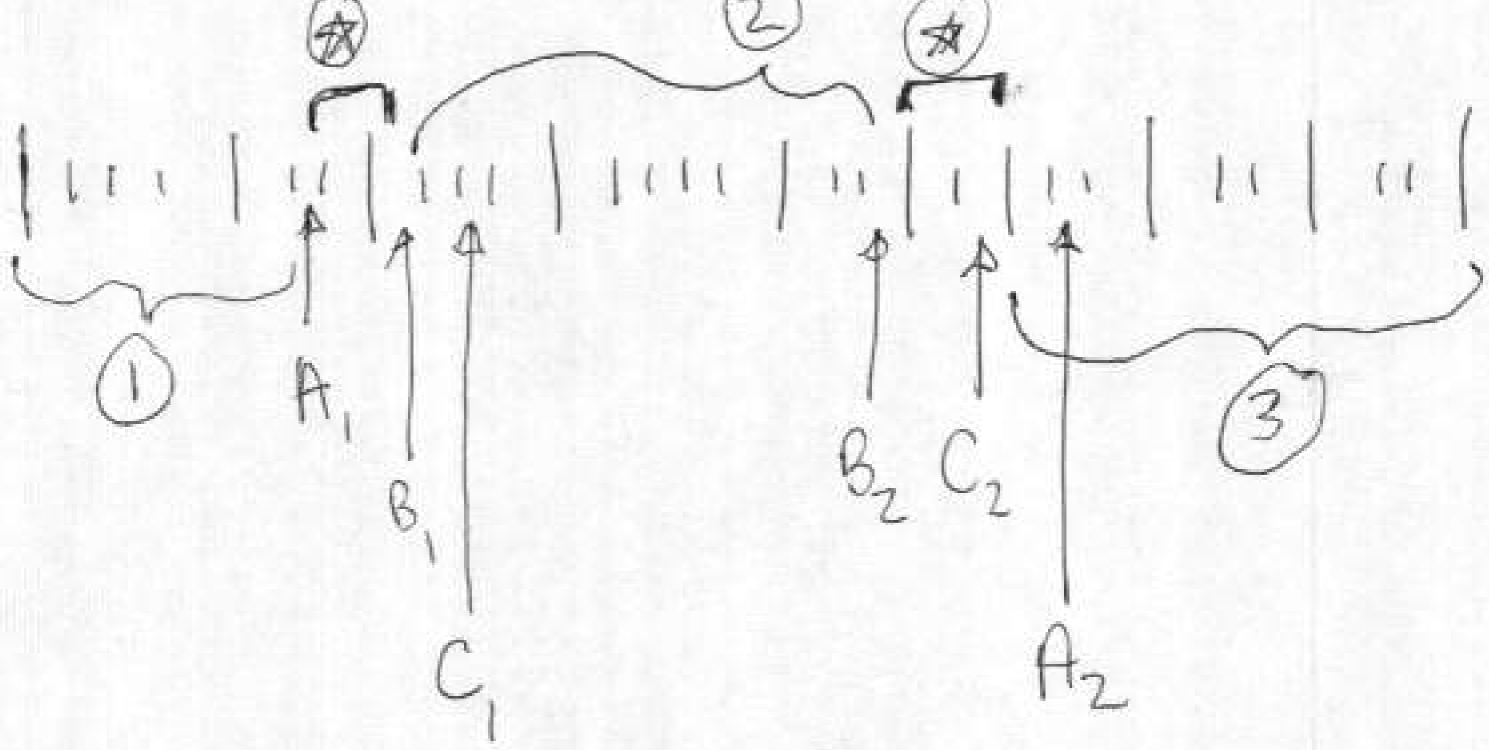
Step 1 (Bidding) Each player writes down independently exactly where he/she wants to place 2 markers. This is done in such a way that the player is OK taking any of the resulting divisions. For example, if player A places markers as follows:



then player A sees if fair to get any one of the three divisions labelled 1, 2, 3.

Step 2 (Allocations) We scan the array from left to right looking for the first "first marker".

We then give all the candy left of that marker to the owner of the marker. For example,



1 goes to A. Then we scan onwards for the first "second marker," i.e. one of A_2, B_2, C_2.
The owner of that marker gets all of the candy between his/her first marker and second marker. In the above example, 2 goes to B. Finally, the remaining player (C in the example) gets all the candy from the right side of his/her second marker. So 3 goes to C.

<u>Step 3</u> (Division of the surplus) There's leftover candy! We just divide this as equally as possible, by flipping coins and having the players choose pieces. (Above, ★ = surplus.)