When looking at a sequential coalition, we can determine the first player (from left to right) that makes the coalition a winning coalition.

For example, in the system $[51 : 49, 48, 3]$, the sequential coalition

$$\langle P_1, P_2, P_3 \rangle$$

has P_1 first, but P_1 with its 49 votes does not alone make a winning coalition. The second player, P_2, when added to P_1, creates a coalition of 97 votes, which is a winning coalition. Thus P_2 is the player that *first* makes $\langle P_1, P_2, P_3 \rangle$ a winning coalition.

The first player that makes a sequential coalition a winning coalition is called a <u>pivotal player</u> in that coalition.

Above, we saw that P_2 was ~~a~~ the pivotal player for the sequential coalition $\langle P_1, P_2, P_3 \rangle$.

6

The <u>pivotal count</u> for a player, written SS_1 for P_1, is the total number of times that the player is a pivotal player when going over all sequential coalitions.

We rewrite the sequential coalitions for $[51: 49, 48, 3]$ and underline the pivotal players:

$$\langle P_1, \underline{P_2}, P_3 \rangle$$

$$\langle P_1, \underline{P_3}, P_2 \rangle$$

$$\langle P_2, \underline{P_1}, P_3 \rangle$$

$$\langle P_2, \underline{P_3}, P_1 \rangle$$

$$\langle P_3, \underline{P_1}, P_2 \rangle$$

$$\langle P_3, \underline{P_2}, P_1 \rangle$$

Pivotal counts: $SS_1 = 2, \ SS_2 = 2, \ SS_3 = 2.$

The Shapley-Shubik Power Index (SSPI) of a player is the ratio of the player's critical count over the total pivotal count of all players, which in a system of N players is equal to $N!$ as we saw below. The SSPI of P_1, for instance, is written $\sigma_1 = SS_1/N!$.

The Shapley-Shubik Power Distribution of a weighted voting system is the list of SSPI's of all the players: $\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_N$.

For $[51: 49, 48, 3]$, (here $N=3$ so $N! = 6$)

$$\sigma_1 = \frac{SS_1}{N!} = \frac{2}{6} = \frac{1}{3}$$

$$\sigma_2 = \frac{SS_2}{N!} = \frac{2}{6} = \frac{1}{3}$$

$$\sigma_3 = \frac{SS_3}{N!} = \frac{2}{6} = \frac{1}{3}$$

So for $[51 : 49, 48, 3]$ the Banzhaf Power distribution and the Shapley-Shubik Power distribution are the same.

Let's do $[4 : 3, 2, 1]$.

First we list the sequential coalitions.

$$\langle P_1, \underline{P_2}, P_3 \rangle$$
$$\langle P_1, \underline{P_3}, P_2 \rangle$$
$$\langle P_2, \underline{P_1}, P_3 \rangle$$
$$\langle P_2, P_3, \underline{P_1} \rangle$$
$$\langle P_3, \underline{P_1}, P_2 \rangle$$
$$\langle P_3, P_2, \underline{P_1} \rangle$$

We then underline the pivotal players.

The pivotal counts are

$$SS_1 = 4, \quad SS_2 = 1, \quad SS_3 = 1.$$

Again $N = 3$ so $N! = 6$ so the SSPI's are

$$\sigma_1 = \frac{SS_1}{N!} = \frac{4}{6} = \frac{2}{3}, \quad \sigma_2 = \frac{SS_2}{N!} = \frac{1}{6}, \quad \sigma_3 = \frac{SS_3}{N!} = \frac{1}{6}$$