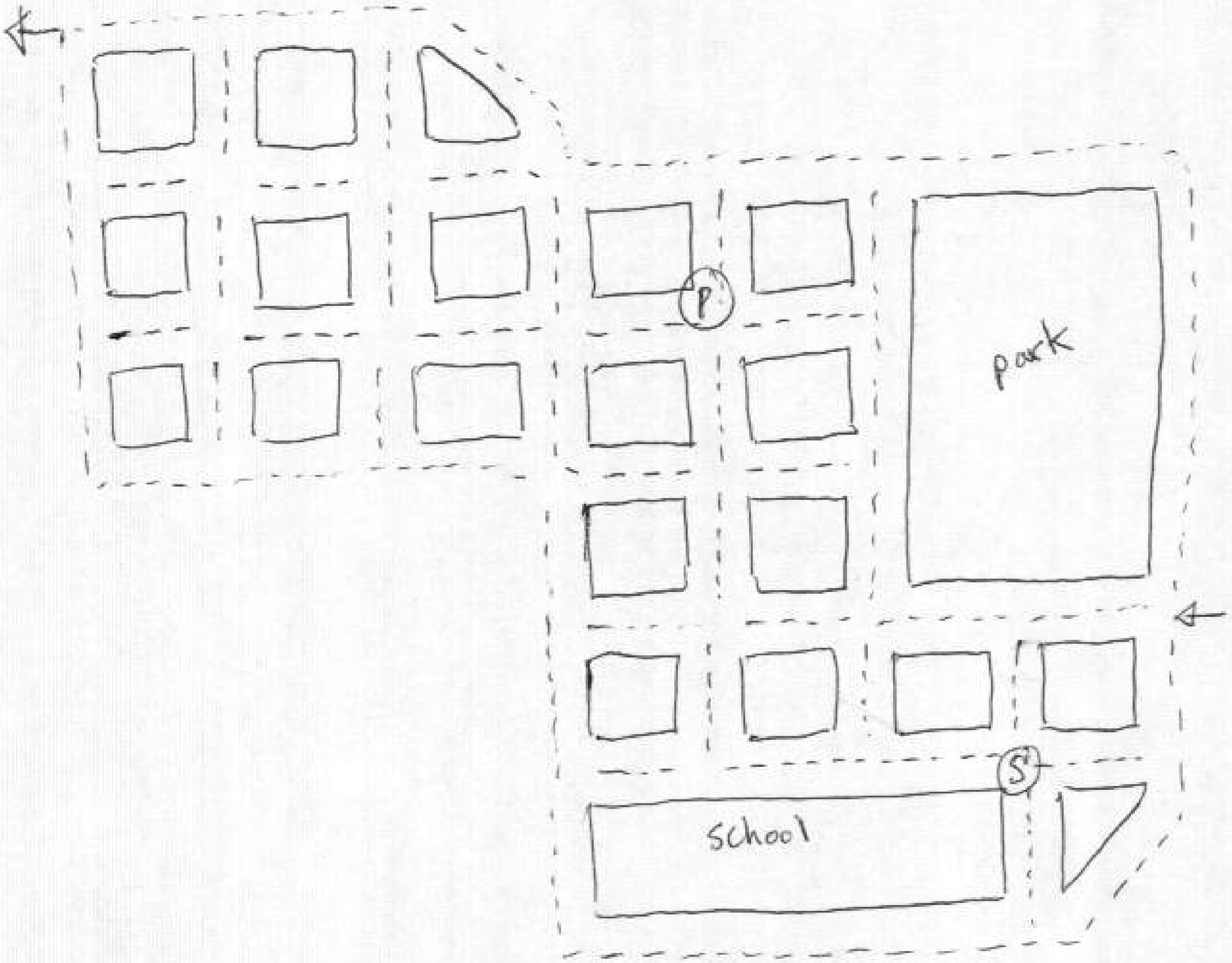
Ch.5 The Mathematics of getting around

Consider the following map: "Sunnyside neighborhood"



The security guard problem:

1. Is it possible for a security guard, walking on foot, to start and end at a given destination (say "S"), in a way such that every block is covered exactly once?

2. If some blocks need to be covered more than once, what is an <u>optimal</u> route, i.e. a route with the least amount of walking?

3. If we don't require the guard to start and end at the same location, can we find a better route?

<u>The mail carrier problem:</u>

This is similar to the security guard problem, but the mail carrier must pass each block <u>twice</u>, once on each side of the street (at least for blocks with buildings on both sides of the street). Also, the mail carrier should begin and end at the post office.

(labelled as (P) on the map.)

The UPS driver problem:

The difference for the UPS driver is that he/she only needs to go to certain designated blocks (ones that have deliveries scheduled). Unlike the mail carrier, we don't require the UPS driver to go down any block more than once (the driver can walk to either side of the street, parking the truck). Finally, the driver needs to enter and exit at the arrows drawn on the map.

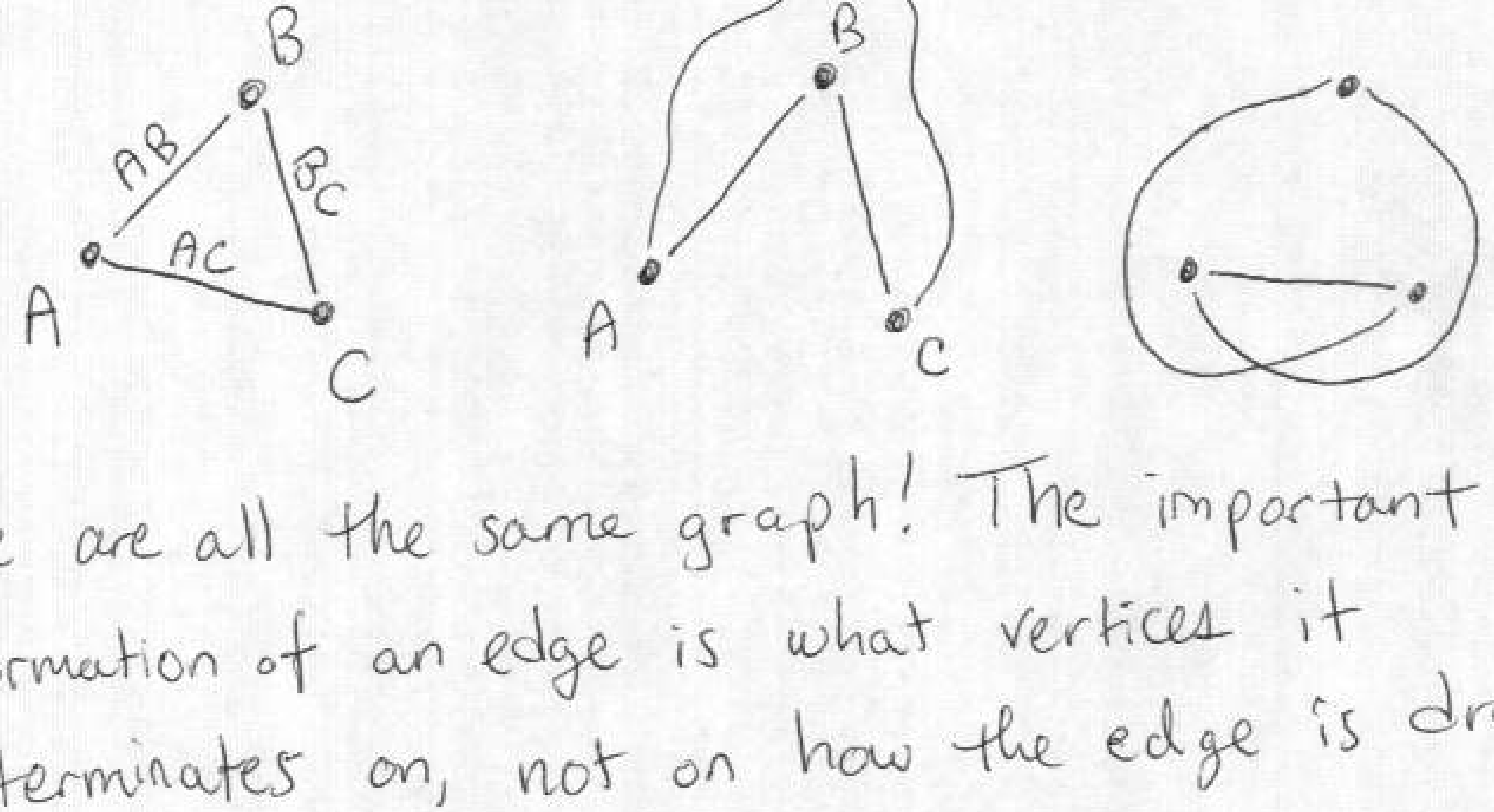
The above problems are examples of Street-routing problems. We will learn how to solve these problems using Graph theory.

We begin by introducing the basic concepts of graph theory.

Definition: A graph is the following data:

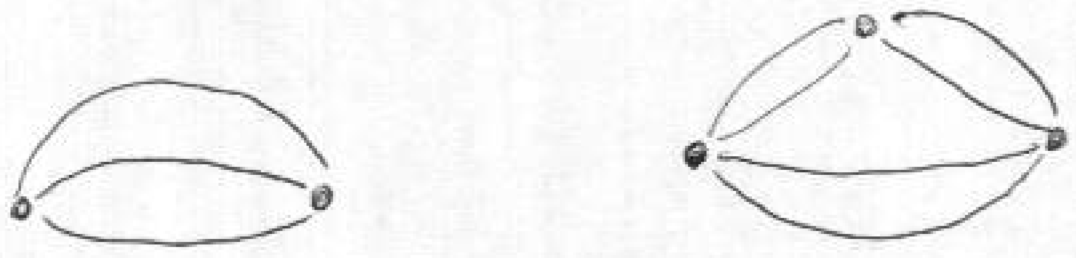
- a set of vertices ("dots") that are usually labelled A, B, C, D, ...
- a set of edges ("lines"). Each edge goes from one vertex to another vertex. An edge going from A to B is usually labelled AB, or BA (direction doesn't matter)

Examples of graphs:



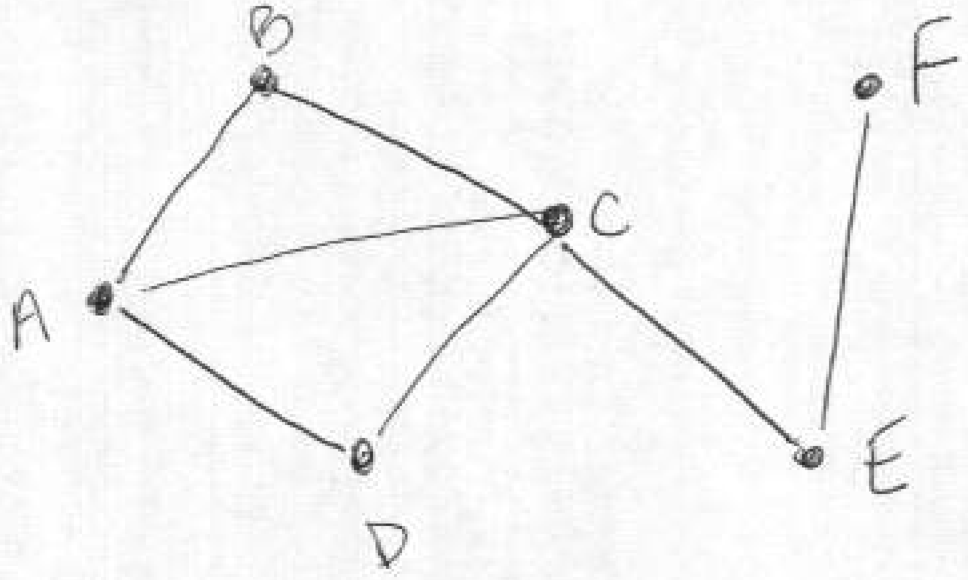
These are all the same graph! The important information of an edge is what vertices it terminates on, not on how the edge is drawn.

Note that we sometimes omit the labels of vertices and/or edges; the labels are useful when we want to distinguish parts in making arguments. We allow <u>multiple edges</u> between vertices, for example:



are valid graphs.

When the vertices of a graph are labelled A, B, ... and we are given a picture of the graph, it is possible to write the complete <u>edge list</u> of the graph. For example:



edge list:

AB, AD, DC, BC, AC, CE, EF

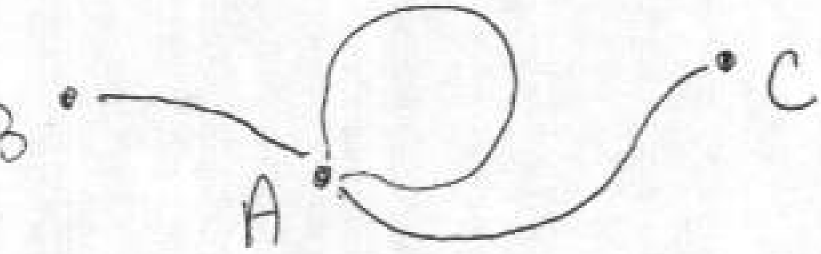
The *degree* of a vertex in a graph is the number of edges that meet at the vertex.
We write $\deg(X)$ for the degree of a vertex X.

For example, in the previously drawn graph,

$$\deg(A) = 3, \quad \deg(B) = \deg(D) = \deg(E) = 2$$

$$\deg(C) = 4, \quad \deg(F) = 1$$

If an edge begins and ends on the same vertex it is called a *loop*.



A loop contributes 2 to the degree of a vertex.
For example, right above, we have

$$\deg(A) = 4.$$