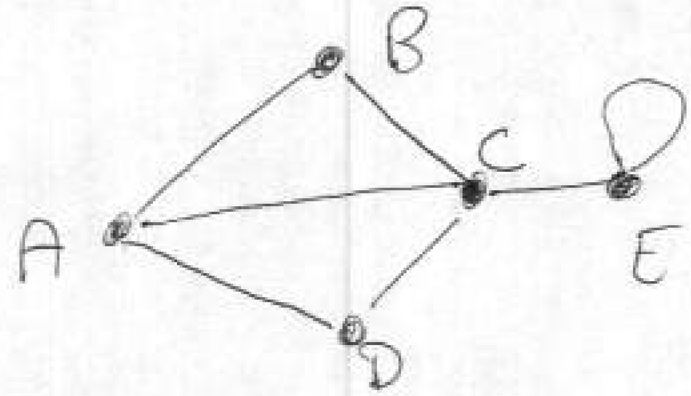
Graph theory continued

Recall the degree of a vertex:



deg(A) = 3

deg(B) = deg(D) = 2

deg(C) = 4

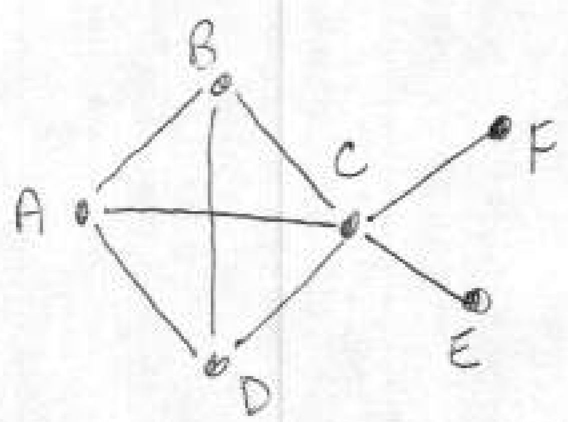
deg(E) = 3

An <u>even</u> (resp. <u>odd</u>) <u>vertex</u> is a vertex with even (resp. odd) degree.

In the above graph: { even vertices: B, C, D; odd vertices: A, E

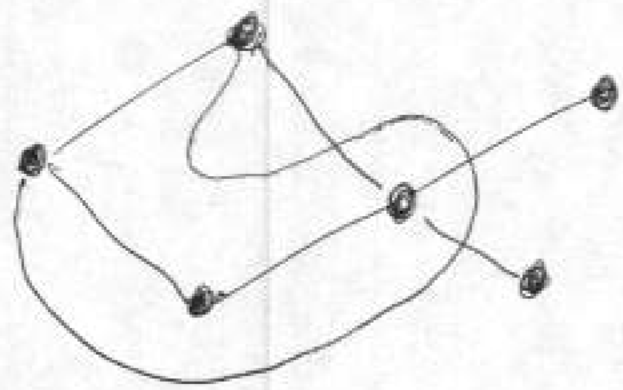
A part of a graph consisting of vertices such that any two of the vertices are connected by an edge is called a <u>clique</u>.

e.g.



A, B, C, D form a clique

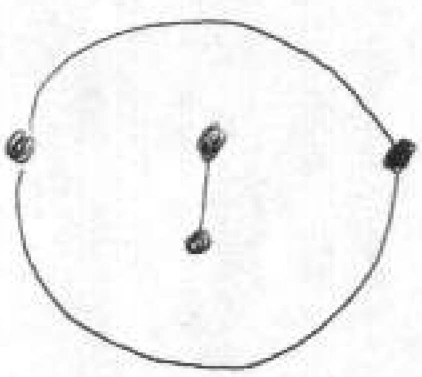
A graph with no multiple edges or loops is called a simple graph. For example, the previous graph is simple, but the first one we looked at is not simple, because it has a loop.



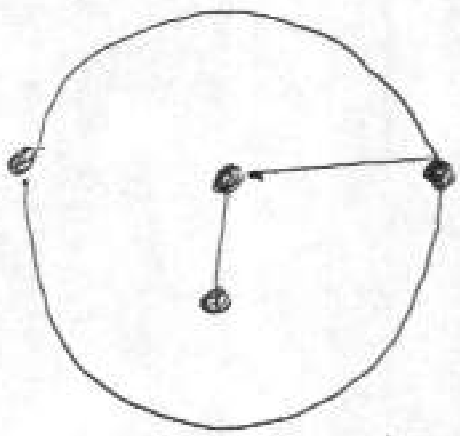
simple?

No. There's a multiple edge

A graph is connected if any two vertices can be joined by a sequence of edges. e.g.,



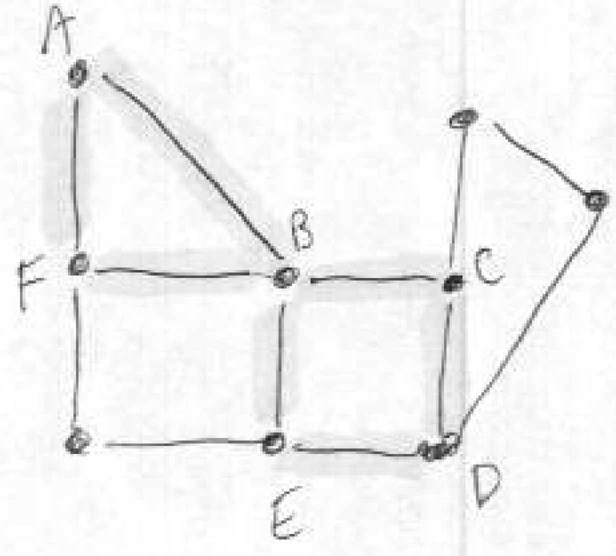
disconnected



connected

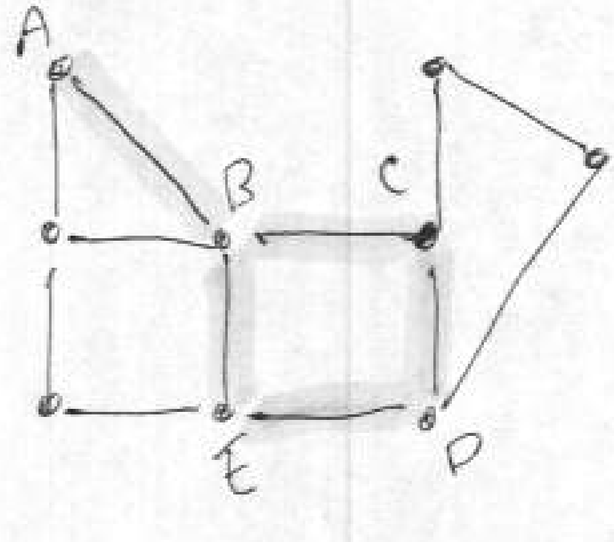
Two edges in a graph are adjacent if they share a vertex. A sequence of distinct edges, each adjacent to the next, is a path.

The number of edges in a path is the length of the path.



The sequence of edges AB, BC, CD, DE, EB, BF, FA forms a path of length 7.

Note that it is OK for a path to revisit vertices, but NOT OK to revisit edges. For example, the following is not considered a path:



AB, BC, CD, DE, EB, BA

This is not a path because the edge AB was passed over twice.

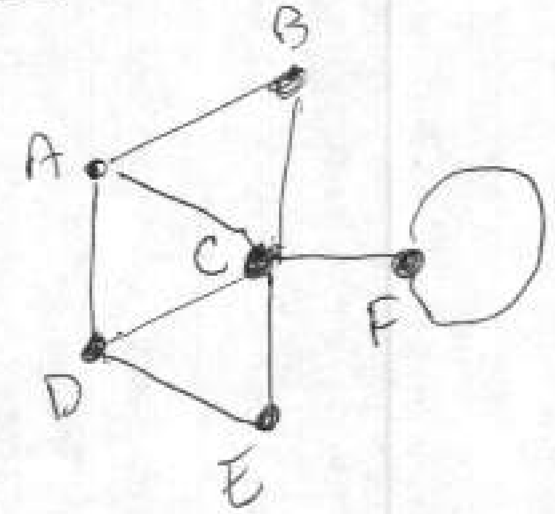
The sequence AB, BC, CD, DE, EB ending at B is a path

In particular, it is a path of length 5.

It is easier to describe a path by listing the vertices: for example, the path of length 7 above is A, B, C, D, E, B, F, A

while the path of length 5 above may be written A, B, C, D, E, B.

Another example:

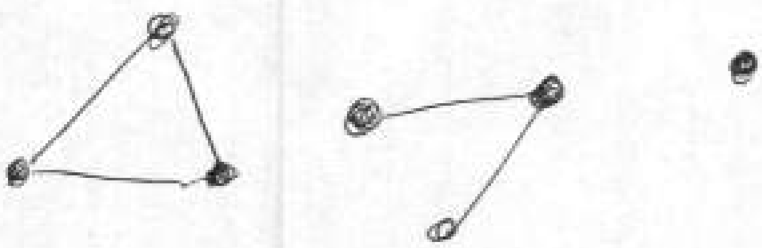


A, D, E, C, F, F is a path of length 5.

A path that begins and ends on the same vertex is called a circuit. In the above graph, F, F (the loop) describes a circuit of length 1. The sequence C, F, F, C is not a circuit, since it goes over the edge CF twice.

We can say that a graph is connected if given any two vertices in the graph, there is a path between them.

The maximally connected parts of a graph are called the components of the graph. I.e.



this graph has 3 components. Thus a connected graph has 1 component. When a vertex has no edges touching it (as above) we call it an isolated vertex. Equivalently, a vertex is isolated if and only if its degree is 0.

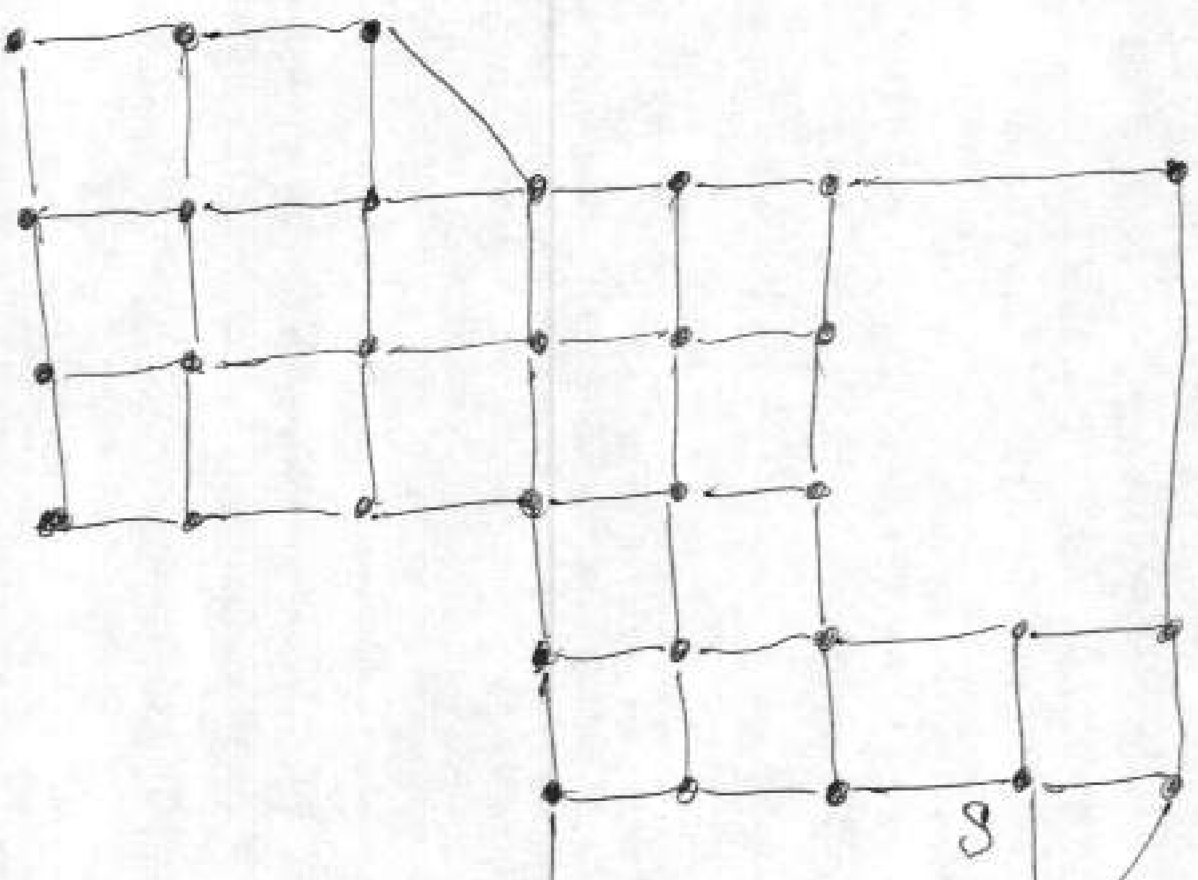
Remember that in the mail carrier problem and the security guard problem we wanted to find routes that covered every block, exactly once in the security guard problem, and sometimes twice in the mail carrier problem.

We make a definition for graphs related to this:

An <u>Euler path</u> in a graph is a path that covers all edges of the graph.

An <u>Euler circuit</u> is a circuit that covers all edges of the graph.

Modelling the security guard problem using graph theory:



The main problem may be stated using our new jargon: does the above graph have any Euler circuits (beginning at vertex S)?