MAT331 Exercises, Fall 2019

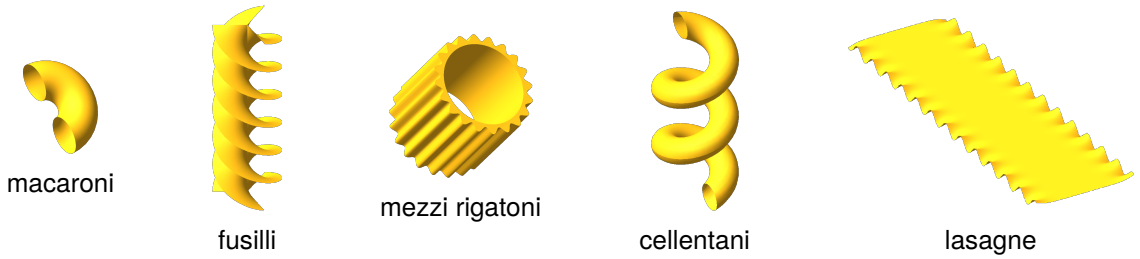
set number 2

5. (*expires 9/16*) Consider the planar curve γ defined by $x^2y^3 + y^2 + y - 2e^x = 0$. Using **only** Maple, find the slope of the tangent line to the curve at $(0, 1)$. Then plot the curve and the tangent line on the same graph.
   Hint: you might want to use `implicitplot` *and* `display` *from the* `plots` *library. You might find* `implicitdiff` *helpful, too.*

6. (*expires 9/16*) Define a Maple function g that, given a positive integer k yields the sum of the first k primes. What is k such that $g(k) \leq 100,000$ but $g(k+1) > 100,000$?
   You might find `ithprime` helpful, and probably `add` (rather than `sum`).

7. (*expires 9/16*) Write a function that, when given a positive integer n as input, will return the n^{th} digit of π (where 3 is viewed as the 0^{th} digit of π, and 4 is the 2^{nd} digit). What is the 2019^{th} digit of π?
   Using `floor` might be helpful, but you could have other ideas.

8. (*expires 9/16*) Use Maple to make pictures of the following pasta.



Here are some relevant equations, in no particular order.

$$z = \sin(2y)\left(1 - e^{-(x/6)^8}\right) \quad -6 \leq x \leq 6, \quad -20 \leq y \leq 20$$

$$\tau = 1 \quad 0 \leq \phi \leq \pi, \quad -\pi \leq \sigma \leq \pi \quad \text{(toroidal coordinates)}$$

$$x = \left(1 + \frac{\cos(s)}{2}\right)\cos(t) \quad y = \left(1 + \frac{\cos(s)}{2}\right)\sin(t) \quad z = 0.4t + \frac{\sin(s)}{2} \qquad \begin{array}{l} 0 \leq s \leq 2\pi \\ \frac{\pi}{2} \leq t \leq \frac{11\pi}{2} \end{array}$$

$$\left\{\begin{array}{lll} x = r\sin(t) & y = r\cos(t) & z = t/2 \\ x = r\sin\left(t + \frac{2\pi}{3}\right) & y = r\cos\left(t + \frac{2\pi}{3}\right) & z = t/2 \\ x = r\sin\left(t - \frac{2\pi}{3}\right) & y = r\cos\left(t - \frac{2\pi}{3}\right) & z = t/2 \end{array}\right\} \qquad \begin{array}{l} 0 \leq r \leq 1 \\ 0 \leq t \leq 4\pi \end{array}$$

$$6 \leq r \leq 7 + \sin(20\theta)/2, \quad 0 \leq \theta \leq 2\pi, \quad 0 \leq z \leq 14 \qquad \text{(cylindrical coordinates)}$$

To help you get started, the Maple worksheet called pasta.mw draws Mezzi Rigatoni. For full credit, your pasta should look like pasta, with appropriate coloring, viewpoint, smoothness, and lighting. Sauce is optional.